**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted from the academic year 2019-20 & thereafter)**

**SUBJECT CODE : 19MT/PE/FT15**

**M. Sc. DEGREE EXAMINATION, APRIL 2023**
**BRANCH I – MATHEMATICS**
**SECOND SEMESTER**

**COURSE: ELECTIVE**
**PAPER: FUZZY SET THEORY AND APPLICATIONS**
**TIME: 3 HOURS** **MAX MARKS: 100**

## SECTION – A

**ANSWER ANY FIVE QUESTIONS:** **(5 × 2 = 10)**

1. Define Height, Normal and Subnormal of a fuzzy set.
2. Define similarity relation $F(X, Y)$.
3. State the axioms for the functions to form the most general class of fuzzy complements.
4. Write down the multiplication and division arithmetic operations for closed intervals.
5. What are the four modules of fuzzy controllers?

## SECTION – B

**ANSWER ANY FIVE QUESTIONS:** **(5 × 6= 30)**

6. For any $A, B \in F(X)$, prove that ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$.
7. State and Prove first Decomposition Theorem.
8. Prove that every fuzzy complement has at most one equilibrium.
9. Define the basic operation of addition(+) and subtraction(-),of fuzzy numbers $A$ and $B$.

$$A(x) = \begin{cases} 0 & \text{for } x \le -1 \text{ and } x > 1 \\ (x+1)/2 & -1 \le x \le 1 \\ (3-x)/2 & 1 < x \le 3 \end{cases} \text{ and } B(x) = \begin{cases} 0 & \text{for } x \le 1 \text{ and } x > 5 \\ (x-1)/2 & 1 \le x \le 3 \\ (5-x)/2 & 3 < x \le 5 \end{cases}$$

10. Explain in detail (i) Centre of Area Method (ii) Maxima Method (iii) Mean of Maxima Method
11. State the axiomatic skeleton for fuzzy set union.
12. For any $A, B \in F(X)$, then for all $\alpha \in [0,1]$ prove that $A \subseteq B$ iff ${}^{\alpha}A \subseteq {}^{\alpha}B$.

## SECTION – C

**ANSWER ANY THREE QUESTIONS:** **(3 × 20 = 60)**

13. (a) Prove that a fuzzy set $A$ on $\mathbb{R}$ is convex if and only if

$$A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)], \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1].$$

(b) Design a trapezoidal and triangular fuzzy number in the interval [0, 80] that represents the concept of young, middle age and old person.

(10 + 10)

14. (a). Let $f : X \to Y$ be an arbitrary crisp function. Then for any $A \in F(X)$ and $\alpha \in [0, 1]$, prove that

$(i).\ {}^{\alpha+}[f(A)] = f({}^{\alpha+}A)$

$(ii).\ {}^{\alpha}[f(A)] \supseteq f({}^{\alpha}A).$

(b) Find the max-min and max product composition for the following two matrices

$$A = \begin{bmatrix} .3 & .5 & .8 \\ 0 & .7 & .1 \\ .4 & .6 & .5 \end{bmatrix} \text{ and } B = \begin{bmatrix} .9 & .5 & .7 & .7 \\ .3 & .2 & 0 & .9 \\ 1 & 0 & .5 & .5 \end{bmatrix}.$$

(10 + 10)

15. (a) Write down the axioms of fuzzy complement

(b) For all $a, b \in [0, 1]$, prove that $u(a, b) \geq \max(a, b)$. (10 + 10)

16. (a) Prove that MIN[A,MAX(A,B)] = A.

(b) Prove that MIN [A, MAX (B, C)] = MAX [MIN (A, B), MIN (A, C)],where

$$\text{MIN(A,B)} = \sup_{z=\min(x,y)} \min[A(x), B(y)]$$

$$\text{MAX(A,B)} = \sup_{z=\max(x,y)} \min[A(x), B(y)]$$

(10+10)

17. Discuss any one application of fuzzy mathematics in Medical Diagnostics and Pattern Recognition.

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