STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/PE/CI15

M. Sc. DEGREE EXAMINATION, APRIL 2023
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : ELECTIVE
PAPER : CALCULUS OF VARIATION AND INTEGRAL EQUATIONS
TIME : 3hours **MAX. MARKS: 100**

Section – A
Answer ALL questions (5 × 2 = 10)

1. Define Euler's function.
2. Explain refraction of extremals.
3. What is separable kernel.
4. Define Fredholm alternative.
5. Write any two applications of ODE.

Section – B
Answer any FIVE questions (5 × 6 = 30)

6. Find the Euler-Ostrogradsky equation for $I[u(x,y)] = \iint_D \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] dxdy$ where the values of u are prescribed on the boundary Γ of the domain D.
7. Find the shortest distance between the parabola $x^2 = y$ and the straight line $x - y = 5$.
8. Find the extremum of the function $I = \int_{x_1}^{x_2} (y'^2 + z'^2 + 2yz)dx$ with $y(0) = 0, z(0) = 0$ and the point (x_2, y_2, z_2) moves over the fixed plane $x = x_2$.
9. Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 (s+t)g(t)dt$ and find the eigen values.
10. Solve the Fredholm integral equation $g(s) = 1 + \lambda \int_0^1 (1 - 3st)g(t)dt$ and evaluate the resolvent kernel.
11. Describe briefly the convolution integral.
12. Explain Dirac delta function.

Section – C
Answer any THREE question (3 × 20 = 60)

13. Derive the necessary condition for the existence of extremal for the functional $I[y(x)] = \int_a^b F(x, y(x), y'(x))\,dx$ subject to the boundary conditions $y(a) = y_1, y(b) = y_2$ where y_1, y_2 are prescribed at the fixed boundary points a, b and $F(x, y(x), y'(x))$ is three times differentiable. Use the condition, to find the curve with fixed boundary points such that its rotation about the axis of abscissae give rise to a surface of revolution of minimum surface area.
14. Explain briefly one-sided variations.
15. Explain briefly Fredholm theorem and also the different types of integral equation.
16. (a) Derive Volterra integral equation

    (b) Find the Neumann series for the solution of the integral equation

    $g(s) = (1 + s) + \lambda \int_0^s (s - t) g(t) dt$
17. Reduce the boundary value problem to a Fredholm integral equation

    $y'' + \lambda s y = 1, y(0) = 0, y(\ell) = 1$ and derive the concept of Green's function.

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