STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/PC/TO24

M. Sc. DEGREE EXAMINATION, APRIL 2023
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE: CORE
PAPER : TOPOLOGY
TIME: 3 HOURS **MAX MARKS:100**

SECTION-A (5×2=10)

ANSWER ALL THE QUESTIONS

1. Define a Topology
2. Give an example of subspace which is not connected
3. What is meant by Compact Spaces
4. Check whether the identity function from $\mathbb{R}_\ell$ to $\mathbb{R}$ is continuous.
5. Explain Completely regular space

SECTION-B (5 × 6 = 30)

ANSWER ANY FIVE QUESTIONS

6. Let $\mathcal{B}$ and $\mathcal{B}'$ be bases of topologies $\mathcal{T}$ and $\mathcal{T}'$ respectively on X. Then prove the following are equivalent:
   (1) $\mathcal{T}'$ is finer than $\mathcal{T}$
   (2) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$.
7. Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.
8. Prove that every closed subspace of a compact space is compact.
9. If X has a countable basis then prove that
   (a) every open covering of X contains a countable subcollection covering X
   (b) There exists a countable subset of X that is dense in X.
10. Prove that every regular space with a countable basis is normal.

11. If X be a set and $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Then prove that

    (a) Any finite intersection of elements of $\mathcal{D}$ is an element of $\mathcal{D}$,

    (b) If A is a subset of X that intersects every element of $\mathcal{D}$, then A is an element of $\mathcal{D}$.

12. Let X be a set and A be a collection of subsets of X having the finite intersection property. Then prove that there is a collection $\mathcal{D}$ of subsets of X such that $\mathcal{D}$ contains A and $\mathcal{D}$ has the finite intersection property and no collection of subsets of X that properly contains $\mathcal{D}$ has this property.

SECTION-C (3 × 20 = 60)

ANSWER ANY THREE QUESTIONS

13. (a) Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each x in U, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Then prove that $\mathcal{C}$ is a basis for the topology of X.

    (b) Show that the countable collection $\mathcal{B} = \{(a,b)/a < b, a \; and \; b \; rational\}$ is a basis that generates the standard topology on $\mathbb{R}$. (15 + 5)

14. Let X be a topological space. Then show that the following conditions hold:

    (1) ϕ and X are Closed.

    (2) Arbitrary intersection of closed sets are closed.

    (3) Finite union of closed sets are closed.

15. (a) Prove that the image of a connected space under a continuous map is connected.

    (b) State and prove the Lebesgue number lemma. (10+10)

16. State and prove Urysohn metrization theorem.

17. State and prove Tychonoff's Theorem.

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