STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/PC/MI24

M. Sc. DEGREE EXAMINATION, APRIL 2023
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : MEASURE THEORY AND INTEGRATION
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer all the questions: **5 × 2 = 10**

1. Define Lebesgue outer measure of a set *A* in $\mathcal{R}$.
2. Show that $\int_1^\infty \frac{dx}{x} = \infty$.
3. Define the Lebesgue integral of a function *f*.
4. When is a measure absolutely continuous with respect to another measure?
5. Let *f* be a function defined on *X* × *Y*. What are the *x* – and *y* – sections of *f* ?

SECTION – B

Answer any five questions: **5 × 6 = 30**

6. Prove that every interval on the real line is measurable.
7. Let *E* be a measurable set. Then show that, for each *y*, the set *E* + *y* is also measurable and the measures are the same.
8. State and prove Lebesgue monotone convergence theorem.
9. Define σ – finite, complete measure and show that the Lebesgue measure is σ – finite and complete.
10. State and prove the Hahn Decomposition theorem.
11. Prove that measurable sets have measurable sections.
12. Show that measurability and measure remain invariant under rotation in *k* dimensions.

SECTION – C

Answer any three questions: **3 × 20 = 60**

13. (a) Show that there exists a non-measurable set.
    (b) Give an example to show that there exists an uncountable set of measure zero.[12 + 8]
14. Let *f* be a bounded function defined on the finite interval [*a*, *b*], then prove that *f* is Riemann integrable over [*a*, *b*] if and only if it is continuous *a.e.*
15. (a) State and prove Fatou's lemma.
    (b) The function $f(x)$, $0 \le x \le 1$, is defined as follows:
    $f(x) = \begin{cases} 0 & if\ x\ is\ rational \\ n & if\ x\ is\ irrational \end{cases}$ where *n* is the number of zeros immediately after the decimal point, in the representation of *x* on the decimal scale. Show that *f* is measurable and find $\int_0^1 f\,dx$. [12 + 8]
16. (a) State and prove Radon Nikodym Theorem.
    (b) Show that the theorem is true for signed measure also. [15 + 5]
17. State and prove Fubini's theorem.

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