STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE: 19MT/PC/DG44

M.Sc. DEGREE EXAMINATION, APRIL 2023
BRANCH I – MATHEMATICS
FOURTH SEMESTER

TITLE: DIFFERENTIAL GEOMETRY
CORE: CORE
TIME: 3 HOURS **MAX: 100 MARKS**

SECTION – A

Answer all the questions (5 × 2 = 10)

1. Define unit speed curve.
2. When a surface is said to be smooth? Give an example.
3. Show that every isometry is a conformal map.
4. Define principal curvatures.
5. Prove that any geodesic has constant speed.

SECTION – B

Answer any five questions (5 × 6 = 30)

6. Define arc length and calculate it for the catenary $\gamma(t) = (t, cosht)$ starting at the point $(0,1)$.
7. Find the equation of the tangent plane of the surface $\sigma(r,\theta) = (rcosh\theta, rsinh\theta, r^2)$ at $(1,0,1)$.
8. Calculate the first fundamental form of a sphere $\sigma(\theta,\varphi) = (cos\theta\ cos\varphi, cos\theta\ sin\varphi, sin\theta)$.
9. State and prove Meusnier's theorem.
10. Let N be the standard unit normal of a surface patch $\sigma(u,v)$. Then prove that $N_u = a\sigma_u + b\sigma_v$ and $N_v = c\sigma_u + d\sigma_v$ where $\begin{pmatrix} a & c \\ b & d \end{pmatrix} = -\mathcal{F}_I^{-1}\mathcal{F}_{II}$.
11. Find the gaussian curvature, mean curvature and principal curvatures for any surface patch $\sigma(u,v)$.
12. Prove that a curve on a surface is a geodesic if and only if its geodesic curvature is zero everywhere.

SECTION – C
Answer any three questions (3 × 20 = 60)

13. Let $\gamma(t)$ be a regular curve in $\mathbb{R}^3$. Then prove that its curvature is $\kappa = \frac{\|\ddot{\gamma}\times\dot{\gamma}\|}{\|\dot{\gamma}\|^3}$ and its torsion is $\tau = \frac{(\dot{\gamma}\times\ddot{\gamma})\cdot\dddot{\gamma}}{\|\dot{\gamma}\times\ddot{\gamma}\|^2}$.

14. (a) Let U and $\widetilde{U}$ be open subsets of $\mathbb{R}^2$ and let $\sigma: U \to \mathbb{R}^3$ be a regular surface patch. Let $\Phi: \widetilde{U} \to U$ be a bijective smooth map with smooth inverse map $\Phi^{-1}: U \to \widetilde{U}$. Then prove that $\tilde{\sigma} = \sigma \circ \Phi: \widetilde{U} \to \mathbb{R}^3$ is a regular surface patch.
(b) Describe an atlas for the surface obtained by translating a curve. (10+10)

15. Prove that a diffeomorphism $f: S_1 \to S_2$ is conformal if and only if, for any surface patch σ_1 on S_1, the first fundamental forms of σ_1 and $f \circ \sigma_1$ are proportional.

16. (a) State and prove Euler's theorem.
(b) Compute the second fundamental form of the elliptic paraboloid
$\sigma(u, v) = (u, v, u^2 + v^2)$. (10+10)

17. Prove that the gaussian curvature of a surface is preserved by isometrics.

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