STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE: 19MT/PC/CF44

M.Sc. DEGREE EXAMINATION, April 2023
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : CONTINUUM AND FLUID MECHANICS
TIME : 3 HOURS **MAX. MARKS : 100**

Section – A
Answer **ALL** questions $(5 \times 2 = 10)$

1. State any two substitution properties of Kronecker δ.
2. Write the equation of motion in Lagrangian formulation.
3. Define viscous fluid with an example.
4. Discuss the conditions at the boundary of two inviscid immiscible fluids.
5. Is Reynolds number dimensional? Explain it.

Section – B
Answer **ANY FIVE** questions $(5 \times 6 = 30)$

6. Given that $a_{ij} = \alpha \delta_{ij} b_{kk} + \beta b_{ij}$ where $\beta \neq 0$, $3\alpha + \beta \neq 0$, find b_{ij} in terms of a_{ij}.
7. The Lagrangian description of a deformation is given by $x_1 = X_1 + X_3\left(e^2 - 1\right)$, $x_2 = X_2 + X_3\left(e^2 - e^{-2}\right)$, $x_3 = e^2 X_3$, where e is a constant. Show that the Jacobian J does not vanish and determine the Eulerian equations describing this motion.
8. Derive the equation of continuity.
9. Explain the uniform flow past a fixed infinite circular cylinder.
10. Show that $[e_i' \quad e_j' \quad e_k'] = \begin{vmatrix} \alpha_{i1} & \alpha_{i2} & \alpha_{i3} \\ \alpha_{j1} & \alpha_{j2} & \alpha_{j3} \\ \alpha_{k1} & \alpha_{k2} & \alpha_{k3} \end{vmatrix}$ and hence, deduce that $\det \alpha_{ij} = 1$.
11. State and prove uniqueness theorem.
12. Discuss the flow for which $w = z^2$.

Section – C

Answer **ANY THREE** questions $(3 \times 20 = 60)$

13. If a_{ij} are components of an isotropic tensor (of second order), then prove that $a_{ij} = \alpha\,\delta_{ij}$ for some scalar α.

14. The stress tensor values at P are given by the array $\Sigma = \begin{pmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{pmatrix}$. Determine the traction (stress) vector on the plane at P whose unit normal is $n = \left(\frac{2}{3}\right)\hat{e}_1 - \left(\frac{2}{3}\right)\hat{e}_2 + \left(\frac{1}{3}\right)\hat{e}_3$ and for the traction vector find (i) the component perpendicular to the plane (ii) the magnitude of $t_i^{(n)}$ (iii) the angle between $t_i^{(n)}$ and n.

15. Test whether the motion specified by $q = \frac{k^2(x\vec{j} - y\vec{i})}{x^2 + y^2}$ (k = constant) is a possible motion for an incompressible fluid. If so, determine the equations of streamlines. Also test whether the motion is of the potential kind and if so determine the velocity potential.

16. Derive (a) Euler's equation of motion (b) Bernoulli's Equation.

17. Discuss the steady motion between parallel planes.

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