STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE: 19MT/PC/CA44

M.Sc. DEGREE EXAMINATION, April 2023
BRANCH I – MATHEMATICS
FOURTH SEMESTER

TITLE: COMPLEX ANALYSIS
CORE: CORE
TIME: 3 HOURS **MAX: 100 MARKS**

SECTION – A
Answer ALL the questions $(\mathbf{5 \times 2 = 10})$

1. Define index of a point.
2. Define Harmonic function.
3. Define infinite product.
4. State Riemann Mapping theorem.
5. When do you say a function is real analytic?

SECTION – B
Answer ANY FIVE questions $(\mathbf{5 \times 6 = 30})$

6. State and prove Cauchy's integral formula
7. State and prove Schwarz's theorem.
8. Show that $\Gamma(z)\Gamma(1-z) = \pi\ cosec \pi z$ and hence deduce that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
9. Show that the function $\xi(s) = \frac{1}{2}s(1-s)\pi^{-\frac{s}{2}}\,\Gamma\left(\frac{s}{2}\right)\zeta(s)$ is entire and satisfies $\xi(s) = \xi(1-s)$.
10. State and prove Jensen's formula.
11. Prove that a family $\mathfrak{F}$ is normal iff its closure $\overline{\mathfrak{F}}$ with respect to the distance function $\rho(f,g) = \sum_{k=1}^{\infty} \delta_k\,(f,g)2^{-k}$ is compact.
12. Illustrate briefly the use of Schwarz – Christofell transformation in finding the complex potential for the flow of a fluid in a channel with abrupt change in its breadth.

SECTION – C
Answer ANY THREE questions $(\mathbf{3 \times 20 = 60})$

13. a) If $f(z)$ is analytic in an open disc Δ then prove that $\int_{\gamma} f(z)dz = 0$ for all closed curve γ in Δ.
    b) If γ lies inside of a circle then prove that the index $n(\gamma, a) = 0$ for all points 'a' outside of the same circle.
14. a) State and prove the mean value property of Harmonic function.
    b) State and prove Poisson's formula for harmonic function.
15. State and prove Mittag – Leffler theorem.
16. State and prove Arzela's theorem .
17. State and prove Schwarz- Christofell formula.

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