STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/MC/SS44

B. Sc. DEGREE EXAMINATION, APRIL 2023
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : SEQUENCE AND SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY TEN QUESTIONS: **(10×2=20)**

1. Define characteristic function of a subset A.
2. Prove that if B is a countable subset of uncountable set A, then $A - B$ is uncountable.
3. Define convergent sequence.
4. What is a Monotone sequence and give an example.
5. Define Cauchy sequence.
6. Differentiate conditional convergence and absolute convergence of series.
7. Explain domination of series with an example.
8. State Abel's lemma.
9. Find the limit superior and limit inferior of $\left\{(1+\frac{1}{n})^n\right\}_{n=1}^{\infty}$.
10. Evaluate $\lim\limits_{n\to\infty} \sqrt{n}(\sqrt{n+1}-\sqrt{n})$.
11. State the Dirichlet's conditions for Fourier series expansion.
12. Write the half range cosine series of $f(x)$.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5×8=40)**

13. Show that the inverse image of the union of two sets is the union of the inverse images.
14. Prove that the set $[0,1] = [x/0 \leq x \leq 1]$ is uncountable.
15. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequence of real numbers, if $\lim\limits_{n\to\infty} s_n = L$ and $\lim\limits_{n\to\infty} t_n = M$.

    Then prove that a) $\lim\limits_{n\to\infty}(s_n + t_n) = L + M$ b) $\lim\limits_{n\to\infty}(s_n t_n) = LM$
16. Show that any bounded sequence of real numbers has a convergent subsequence.

17. If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive real numbers such that

(a) $a_1 \geq a_2 \geq a_3 \ldots a_n \geq a_{n+1} \ldots$ and

(b) $\lim_{n\to\infty} a_n = 0$ then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ is convergent.

18. Show that the series $\sum_{n=1}^{\infty}\frac{1}{n}$ is divergent.

19. Obtain the Fourier series expansion of $xsinx \ in \ [-\pi, \pi]$

SECTION – C

ANSWER ANY TWO QUESTIONS: (2×20=40)

20. a) If $A_1, A_2, A_3 \ldots$ are countable then prove that $\cup_{n=1}^{\infty} A_n$ is countable.

b) Show that the sequence $\left\{(1+\frac{1}{n})^n\right\}_{n=1}^{\infty}$ is convergent.

21. a) Prove that the sequence of real numbers$\{s_n\}_{n=1}^{\infty}$ is Cauchy if and only if it is convergent.

b) Let $\sum_{n=1}^{\infty} a_n$ be a series of non-zero real numbers

and let $a = \lim_{n\to\infty} inf \left|\frac{a_{n+1}}{a_n}\right|$, $A = \lim_{n\to\infty} sup \left|\frac{a_{n+1}}{a_n}\right|$. Then prove that,

1) If $A < 1, then \ \sum_{n=1}^{\infty}|a_n| < \infty$;

2) If $a > 1 \ then \ \sum_{n=1}^{\infty} a_n \ diverges$;

3) If $a \leq 1 \leq A, then \ the \ test \ fails.$

22. a) Find the Fourier series expansion of $f(x) = \begin{cases} -\pi & if \ -\pi < x < 0 \\ x & if \ \ 0 < x < \pi \end{cases}$

and deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

b) Expand $\cos x$ in a half range sine series in $0 < x < \pi$.