STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2019-20 Thereafter)

SUBJECT CODE : 19MT/MC/IC23

B. Sc. DEGREE EXAMINATION, APRIL 2023
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : INTEGRAL CALCULUS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY TEN QUESTIONS: **(10 × 2 = 20)**

1. Evaluate: $\int \frac{dx}{\sqrt{x}+\sqrt{1+x}}$.
2. Evaluate: $\int (1 + sin^3\theta) d\theta$.
3. Determine whether the integral $\int_1^\infty \frac{1}{x^2} dx$ is convergent or divergent.
4. Find $\int_2^5 \frac{1}{\sqrt{x-2}} dx$.
5. State comparison theorem.
6. Define Beta function.
7. When *n* is a positive integer, prove that $\Gamma(n) = (n-1)!$.
8. Write the integral $\int_0^{\pi/2} sin^7\theta \, cos^5\theta \, d\theta$ in terms of Gamma functions.
9. Write down the midpoint rule for double integrals.
10. Evaluate: $\int_1^2 \int_0^3 x^2 y \, dx \, dy$.
11. Define triple Riemann sum.
12. What is the use of Jacobian?

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 × 8 = 40)**

13. Evaluate: $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$.
14. Compute $\int_0^1 \ln x \, dx$.
15. Evaluate: $\int_0^1 x^m \left(log \frac{1}{x}\right)^n dx$.
16. Find the volume of the tetrahedron bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$ and $z = 0$.

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17. Evaluate $\int\int\int_E \sqrt{x^2+z^2}\, dV$ where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.

18. Evaluate: $\int \sqrt{(x-3)(7-x)}\, dx$.

19. Show that $\int_0^\infty e^{-x^2}\, dx$ is convergent.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2 × 20 = 40)**

20. (a) Evaluate: $\int_0^{\pi/2} \frac{dx}{9\cos x + 12\sin x}$

    (b) Prove that $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)} = \pi$. **(10+10)**

21. (a) Derive the relation between Beta and Gamma functions.

    (b) Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of Gamma functions and hence evaluate the integral $\int_0^1 x^5(1-x^3)^{10} dx$. **(12+8)**

22. (a) Evaluate $\int\int_R (3x + 4y^2)\, dA$ where R is the region in the upper half-plane bounded by the circle $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

    (b) Use the change of variables $x = u^2 - v^2$, $y = 2uv$ to evaluate the integral $\int\int_R y\, dA$ where R is the region bounded by x-axis and the parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x$. **(10+10)**