STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/MC/DM43

B. Sc. DEGREE EXAMINATION, APRIL 2023
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : DISCRETE MATHEMATICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY TEN QUESTIONS: **(10×2=20)**

1. Write the following statements in symbolic form:
   (i) Roses are red and violets are blue
   (ii) If either Jerry takes calculus or Ken takes sociology then Larry will take English.
2. Construct the truth table for $(\neg P \wedge Q)$.
3. Let $G(x,y)$: x is taller than y. Translate the following into formula: "For any x and for any y, if x is taller than y then it is not true that y is taller than x"
4. Draw Hasse diagram for $(\rho(A), \subseteq)$ where $A = \{a,b,c\}$.
5. Prove that every finite lattice is bounded.
6. Is the poset $(Z^+, /)$ a lattice?
7. If $(B, +, .)$ is a Boolean algebra and if $a, b \in B$, prove that $a + a'b = a + b$.
8. Define Boolean function.
9. Define complete sequential machine.
10. Find the transition diagram of the finite state automaton $M = (I, S, A, s_0, f)$, where $I = \{0,1\}$, $S = \{s_0, s_1, s_2\}$, $A = \{s_2\}$, s_0 is initial state and the transition function f is given by $f(s_0, 0) = s_1$, $f(s_0, 1) = s_0$, $f(s_1, 0) = s_2$, $f(s_1, 1) = s_0$, $f(s_2, 0) = s_2$, $f(s_2, 1) = s_0$.
11. Let $L = \{a^m b^n : m, n > 0\}$ be a language over $A = \{a, b\}$. Find a regular expression r such that $L = L(r)$.
12. Define context-sensitive grammar.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5×8=40)**

13. Prove that $\big(\neg P \wedge (\neg Q \wedge R)\big) \vee \big((Q \wedge R) \vee (P \wedge R)\big) \Leftrightarrow R$ using equivalence laws.
14. State and prove distributive inequalities of Lattice.
15. Let $(L, \leq)$ be a lattice and a, b, $c \in$ L. If $a \leq b \leq c$ then prove that (i) $a \vee b = b \wedge c$ and (ii) $(a \wedge b) \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.

16. Let a be any element of a Boolean algebra B. Then prove that the complement of a is unique.

17. Obtain the product-of-sums canonical forms of the Boolean expression in three variables x_1, x_2, x_3

    (i) $x_1\, x_2$
    (ii) $x_1 + x_2$
    (iii) $(x_1\, x_2)' + x_3$.

18. Construct a finite state automaton that accepts those strings over $\{0,1\}$ for which the last two input symbols are 1.

19. State and prove Pumping Lemma.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2×20=40)**

20. (a) Find the conjunctive normal form and disjunctive normal form for :

    (i) $p \leftrightarrow (\bar{p} \vee \bar{q})$

    (ii) $(p \vee \bar{q}) \longrightarrow q$.

    (b) Let a, b be elements of a Boolean algebra. Then prove that $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$. (10+10)

21. State and prove idempotent, associative, commutative and absorption properties of Lattice.

22. (a) Let L be a set accepted by a non-deterministic finite automaton. Then prove that there exists a deterministic finite automaton that accepts L.

    (b) Find a context-free grammar G which generates the language $L = \{a^n b^n : n > 0\}$.

(12+8)

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