STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019–20 & thereafter)

SUBJECT CODE : 19MT/MC/CA65

B. Sc. DEGREE EXAMINATION, APRIL 2023
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : PRINCIPLES OF COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ANY TEN QUESTIONS: **10 × 2 = 20**

1. Define Analytic function.
2. State whether the function $f(z) = e^x(\cos y + i \sin y)$ is differentiable. Give reasons.
3. What is Branch cut and Branch point?
4. Define Bilinear transformation.
5. State Cauchy-Goursat Theorem.
6. Define Simply connected and Multiply connected domain.
7. Define Conformal mapping.
8. State Laurent's theorem.
9. Define Isolated Singular point.
10. Define Residue.
11. Show that the transformation $w = iz + i$ maps the half plane $x > 0$ onto the half plane $v > 1$.
12. Find the singular points of the function $(z) = \frac{z+1}{z^3(z^2+1)}$.

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 × 8 = 40**

13. Derive the Cauchy-Riemann equations in polar form.
14. Show that the mapping $w = 1/z$ transforms circles and lines into circles and lines.
15. Evaluate $\int_C \frac{\cos z}{z(z^2+8)}\, dz$ where C is the square bounded by the lines $x = \pm 2$ and $y = \pm 2$.
16. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent's series valid for $|z-1| > 1$ and $0 < |z-2| < 1$.

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17. State and Prove Cauchy's residue theorem.
18. Prove that any bilinear transformation preserves cross ratio.
19. State and Prove Rouche's theorem.

SECTION-C

ANSWER ANY TWO QUESTIONS: $2 \times 20 = 40$

20. (a) Derive Cauchy Riemann equation in Cartesian form.

    (b) Discuss the transformation $w = \sin z$.

21. (a) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto $w_1 = 1, w_2 = i, w_3 = -1$.

    (b) State and prove Cauchy's integral formula.

22. (a) State and prove Taylor's Theorem.

    (b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

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