

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/MC/AG24

B. Sc. DEGREE EXAMINATION, APRIL 2023

BRANCH I – MATHEMATICS

SECOND SEMESTER

COURSE : MAJOR CORE

PAPER : ANALYTICAL GEOMETRY

TIME : 3 HOURS

MAX. MARKS : 100

SECTION A

Answer any ten Questions:

$10 \times 2 = 20$

1. Define center of a conic.
2. How do you determine the eccentricity of the conic?
3. Write the formula to find the length of the equi-conjugate diameter of an ellipse.
4. Discuss the angle between the asymptotes?
5. What are the intercepts made by the plane $2x + 3y + 5z + 7 = 0$ on the coordinate plane?
6. Write the condition that two planes are perpendicular?
7. Find the equation of the line through the point $(3, 2, -1)$ and perpendicular to the plane $5x - 4y + 7z - 1 = 0$.
8. What are skew lines? Give an example.
9. Find the center and radius of the sphere $16x^2 + 16y^2 + 16z^2 - 16x - 8y - 16z - 55 = 0$.
10. A right circular cone has three mutually perpendicular generators. Prove that the semi vertical angle of the cone is $\tan^{-1}(\sqrt{2})$.
11. Prove that the following lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar.
12. Define great circle.

SECTION B

Answer Any Five Questions:

$5 \times 8 = 40$

13. Find the center of the conic given by the general second degree equation.
14. Prove that the product of the perpendiculars drawn from any point on a hyperbola to its asymptotes is constant.
15. Find the angle between the planes $2x - y + z = 6$, $x + y + 2z = 3$.
16. Find the image of the point $(2, 3, 5)$ in the plane $2x + y - z + 2 = 0$.
17. Find the equation of the sphere passing through the points $(1, 0, -1)$, $(2, 1, 0)$, $(1, 1, -1)$ and $(1, 1, 1)$.
18. Express in symmetrical form the line $4x + 4y - 5z - 12 = 0$, $8x + 12y - 13z - 32 = 0$.
19. Prove that the equation $2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$ represents a cone whose vertex is $(2, 2, 1)$

SECTION C

Answer Any Two Questions:

 $2 \times 20 = 40$

20. (a) Prove that a general second degree equations will represent a conic.
 (b) Prove that the acute angle between two conjugate diameters of an ellipse is minimum when they are equal.
21. (a) Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-15}{-5}$ and find the equation of the line of shortest distance.
 (b) (i) Find the equation of the plane passing through the points (9, 3, 6) and (2, 2, 1) and perpendicular to the plane $2x+6y+6z-9=0$.
 (ii) Prove that the tangent to a rectangular hyperbola terminated by its asymptotes is bisected at the point of contact and encloses a triangle of constant area.
22. (a) Find the center and radius of the circle $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0, x - 2y + 2z = 3$.
 (b) Find the equation of the right circular cone whose vertex at the origin, whose axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has a vertical angle of 60° .

