

**SUBJECT CODE : 19PH/PC/MP14**

**M.Sc., DEGREE EXAMINATION NOVEMBER 2022**

**PHYSICS**

**FIRST SEMESTER**

**COURSE : MAJOR CORE**

**PAPER : MATHEMATICAL PHYSICS - I**

**TIME : 3 HOURS**

**MAX. MARKS : 100**

**SECTION - A**

**ANSWER ALL QUESTIONS:**

**(10x3=30)**

1. What do you mean by interpolation and extrapolation?
2. Find  $y(0.1)$  for  $y' = \frac{x-y}{2}$ ,  $y(0) = 1$  with step length 0.1 using Runge –Kutta method of second order.
3. Verify Cauchy – Riemann equations for the following function  $f(z) = az + b$ .
4. Find the pole of the following function  $\frac{z}{(z-1)(z-2)^2}$
5. What is unitary space? Give example.
6. Define isomorphism of vector space.
7. What is Einstein's summation convention?
8. If  $A_{ij}$  is antisymmetric tensor, find the component  $A_{11}$ .
9. Define Beta function.
10. What is Hankel function?

**SECTION – B**

**ANSWER ANY FIVE QUESTIONS:**

**(5x5=25)**

11. Using Newton Raphson method, obtain a root of  $x^3-3x-5 = 0$  to three decimals
12. Using 4 segment Simpsons 1/3 rule to approximate the distance covered by a rocket in meters from  $t=8s$  to  $t = 30s$  given by
$$x = \int_8^{30} 2000 \ln \left( \frac{14000}{14000-2100t} \right) - 9.8t. dt$$
13. Prove that the function  $u = 2x - x^3 + 3xy^2$  satisfies Laplace equation and determine the imaginary part of the corresponding regular function  $u+iv$ .
14. Expand  $f(z) = \sin z$  into a Taylor series about  $z = \pi/4$
15. Derive Fourier's equation of heat flow in solids.
16. Obtain moment of inertia in tensor form.
17. By using Rodrigue's formula, find first four Legendre polynomials.

**SECTION – C**

**ANSWER ANY THREE QUESTIONS:**

**(3x15=45)**

18. Find the numerical solution of  $\frac{dy}{dx} = x + y$ , from  $x=0$  to  $0.2$  by modified Euler's method for  $h= 0.05$  with the initial condition  $x_0 = 0, y_0 = 1$ .
19. (a) State and prove Cauchy's integral formula. (10 mark)  
(b) Evaluate  $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ , where  $C$  is the circle  $|z| = 3$ , using Cauchy's integral formula.

20. Use Gram Schmidt orthogonal process to construct an orthonormalization to the following basis for  $\mathbb{R}^3$

$$B = \{(1,1,0), (1,2,0), (0,1,2)\}$$

21. (a) If  $A^\mu$  and  $B_\mu$  are any two vectors, one contravariant and the other covariant, then prove that  $A^\mu B_\mu$  is invariant.  
(b) A covariant tensor has components  $xy$ ,  $2y - z^2$ ,  $xz$  in rectangular coordinates. Find its covariant components in spherical coordinates
22. Obtain the complete solution for Bessel differential equation.

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