STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20)

SUBJECT CODE : 19MT/PC/RA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 × 2 = 10)**
ANSWER ALL QUESTIONS

1. Define accumulation point and find the accumulation points of set of integers.
2. Define function of a bounded variation and give an example of a function which is bounded but not of bounded variation.
3. Give an example of a function which is not Riemann-Stieltjes integrable.
4. Write the Jacobian matrix for total derivative of function $\bar{f} : S \subset R^n \to R^m$
5. Define stationary point and saddle point

SECTION – B **(5 × 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. Show that a set S is closed in R^n if and only if it contains all its adherent points.
7. State and Prove Cantor intersection theorem.
8. Let f be a function of bounded variation on $[a,b]$. Let $V(x) = V_f(a,x)$ for $a < x \le b$ and $V(a) = 0$. Show that V and $V - f$ are increasing functions on $[a,b]$.
9. Establish the linear property for two integrators with respect to Riemann-Stieltjes integrals
10. State and prove the sufficient conditions for existence of Riemann-Stieltjes integral
11. State and prove the mean value theorem for a function of several variables.
12. State and prove the sufficient condition for a function f to have a local extremum at a point c

SECTION – C **(3 × 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. (a) State and Prove Lindelof covering theorem. (10)

(b) Let S be a subset of R^n. Show that if any infinite subset of S has an accumulation point in S, then S is compact. (10)

14. (a) If f is a bounded variation on $[a, b]$ and $c \in (a,b)$ then show that

$V_f(a,b) = V_f(a,c) + V_f(c,b)$ (10)

(b) State and prove Integration by Parts formula for Riemann-Stieltjes integrals (10)

15. (a) Assume that $\alpha \uparrow$ on $[a,b]$. Show that the following three statements are equivalent

(i) $f \in R(\alpha)$ on $[a,b]$

(ii) satisfies Riemann's condition with respect to α on $[a,b]$

(iii) $\underline{I}(f,\alpha) = \bar{I}(f,\alpha)$ (15)

(b) Assume that $\alpha \uparrow$ on $[a,b]$. If $f \in R(\alpha)$ on $[a,b]$, show that $f^2 \in R(\alpha)$ on $[a,b]$ (5)

16. (a) State and prove the sufficient conditions for equality of mixed partial derivatives of function of several variables $\bar{f}$ at a given point $\bar{c}$. (15)

(b) Justify with an example of a function whose mixed partial derivatives exist a given point but they are unequal. (5)

17. State and prove Inverse function theorem

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