STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 and thereafter)

SUBJECT CODE : 19MT/PC/PD34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : PARTIAL DIFFERENTIAL EQUATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 × 2 = 10)**

1. Write the three classes of integrals of Partial differential equation.
2. Classify the following differential equation.
   $(1+x^2)u_{xx} + (1+y^2)u_{yy} + xu_x + yu_y = 0.$
3. State the Dirichlet Problem of a rectangle.
4. Define Dirac delta function.
5. What is wave function?

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 × 6 = 30)**

6. Find the general solution of the differential equation $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z$.
7. Find a complete integral of the equation $p^2x + q^2y = z$.
8. Derive the Laplace Equation.
9. Solve the one-dimensional diffusion equation in the region $0 \le x \le \pi, t \ge 0$, subject to the conditions (i) T remains finite as $t \to \infty$ (ii) $T = 0, if\ x = 0\ and\ \pi\ for\ all\ t$
   (iii) $t = 0, T = \begin{cases} x, & 0 \le x \le \pi/2 \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}$.
10. Derive the D'Alembert's solution for one-dimensional wave equation.
11. Derive the Poisson equation.
12. A uniform rod of length L whose surface is thermally insulted is initially at temperature $\theta = \theta_0$. At time $t = 0$, one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution $\theta(x,t)$.

SECTION – C

ANSWER ANY THREE QUESTIONS: (3 × 20 = 60)

13. (a) Find the integral surface of the linear partial differential equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0, z=1$.
   (b) Find the solution of the equation $z = \frac{1}{2}(p^2+q^2) + (p-x)(q-y)$ which passes through the x-axis.

14. (a) Find a complete integral of the partial differential equation $(p^2+q^2)x = pz$ and deduce the solution which passes through the curve $x=0, z^2=4y$
   (b) Transform the following differential equation to a canonical form $u_{xx} + 2u_{xy} + 4u_{yy} + 2u_x + 3u_y = 0$.

15. (a) Discuss the Neumann problem for a rectangle and find its solution.
   (b) Discuss the Exterior Dirichlet problem for a circle and find its solution.

16. (a) Let $f(t)$ be any continuous function. Then prove that $\int_{-\infty}^{\infty} \delta(t-a)f(t)dt = f(a)$.
   (b) Find the solution of the one-dimensional diffusion equation satisfying the following BCs:
   (i) T is bounded as $t \to \infty$ (ii) $\left[\frac{\partial T}{\partial x}\right]_{x=0} = 0, for\ all\ t$
   (iii) $\left[\frac{\partial T}{\partial x}\right]_{x=a} = 0, for\ all\ t$ (iv) $T(x,0) = x(a-x), 0 < x < a.$

17. (a) Solve the following initial value problem of the wave equation (Cauchy problem), described by the inhomogeneous wave equation
   PDE: $u_{tt} - c^2 u_{xx} = f(x,t)$ subject to the initial conditions
   $u(x,0) = \eta(x);\ \ u_t(x,0) = v(x)$.
   (b) Obtain the solution of the wave equation $u_{tt} = c^2 u_{xx}$ under the following conditions:
   (i) $u(0,t) = u(2,t) = 0$ (ii) $u(x,0) = sin^3(\frac{\pi x}{2})$ (iii) $u_t(x,0) = 0$

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