STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE: 19MT/PC/OD14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**ORDINARY DIFFERENTIAL EQUATIONS**
TIME	**:**	**3 HOURS** **MAX. MARKS : 100**

SECTION – A **(5 × 2 = 10)**
ANSWER ALL THE QUESTIONS

1. Determine whether the functions $\phi_1(x) = \cos x$, $\phi_2(x) = \sin x$ are linearly independent on $-\infty < x < \infty$
2. Define Fundamental matrix.
3. Find the singular points of the equation $tx'' + 4x = 0$.
4. Define Lipschitz condition.
5. Explain Sturm-Liouville problem.

SECTION – B **(5 × 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. Compute the Wronskian of $\phi_1(x) = 1$ $\phi_2(x) = \cos x$ and $\phi_3(x) = \sin x$
7. Determine a fundamental matrix for $x' = Ax$,where $A = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$
8. Show that $\int_{-1}^{1} P_n^{\,2}(x)dx = \dfrac{2}{2n+1}$
9. Compute the successive approximations x_n, $n \geq 0$ for the IVP $x' = -x, x(0) = 1, t \geq 0$
10. Find the eigenvalues and eigenvectors of the BVP $x'' + \lambda x = 0, 0 < t \leq \pi \quad , x'(0) = 0, x'(\pi) = 0$
11. Show that $x^{1/2} J_{1/2}(x) = \dfrac{\sqrt{2}}{\Gamma\left(\dfrac{1}{2}\right)} \sin x$
12. Prove that a solution matrix Φ of $X' = A(t)X, t \in I$ on I is a fundamental matrix of $x' = A(t)x, t \in I$ on I if and only if $\det \Phi \neq 0$ for $t \in I$

SECTION – C **($3 \times 20 = 60$)**

ANSWER ANY THREE QUESTIONS

13. (a) If x_1 , x_2 are linearly independent solutions of the equation L(x) = 0 on I then prove that Wronskian of x_1 and x_2 namely $W[x_1(t), x_2(t)]$ is never zero on I.

(b) Suppose that z_1 is a solution of $L(y) = d_1$ and that z_2 is a solution of $L(y) = d_2$. Then show that $z_1 + z_2$ is a solution of the equation $L(y)(t) = d_1(t) + d_2(t)$.

14. (a) If $\Phi(t), t \in I$ be a fundamental matrix of the system $x' = Ax$ such that $\Phi(0) = E$, where A is a constant matrix. Here E denote the identity matrix then show that Φ satisfies $\Phi(t+s) = \Phi(t)\Phi(s)$ for all values of t and $s \in R$

(b) Consider the system $x' = Ax + b(t)$, where $A = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}$ and for $t > 0$, $b(t) = \begin{bmatrix} e^t \\ e^{-t} \end{bmatrix}$. Show that $\Phi(t) = \begin{bmatrix} e^{3t} & 2te^{3t} \\ 0 & e^{3t} \end{bmatrix}$ is a fundamental matrix of $x' = Ax$.

15. Compute the indicial polynomial and their roots for the equation $x^2 y'' + (x + x^2) y' - y = 0$.

16. State and prove Picard's theorem.

17. If $G(t, s)$ be Green's function then prove that $x(t)$ is a solution of $L(x) + f(t) = 0, a \le t \le b$ if and only if $x(t) = \int_a^b G(t,s) f(s) ds$

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