STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20)

SUBJECT CODE: 19MT/PC/GT14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : GRAPH THEORY
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 X 2 = 10)**

ANSWER ALL QUESTIONS

1. Define automorphism of a graph.
2. Prove that if *G* is a *k*-regular bipartite graph with $k > 0$, then *G* has a perfect matching.
3. In a critical graph, Prove that no vertex cut is a clique.
4. Define dominating and independent dominating sets.
5. Define dilation of the embedding $\emptyset$.

SECTION – B **(5 X 6 = 30)**

ANSWER ANY FIVE QUESTIONS

6. Prove that a vertex *v* of a tree *G* is a cut vertex of *G* if and only if $d(v) > 1$.
7. With usual notations, Prove that $\alpha' + \beta' = v,\ \ \delta > 0$.
8. Prove that if *G* is a simple graph, $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$ for any edge *e* of *G*.
9. If *G* is connected plane graph, Prove that $v - \varepsilon + \emptyset = 2$ where $v, \varepsilon, \emptyset$ represent the number of vertices, edges and faces of *G* respectively.
10. Explain some basic principles of network design.
11. Let *G* be a connected graph that is not an odd cycle. Then prove that *G* has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.
12. Let *G* be a nonplanar connected graph that contains no subdivision of K_5 or $K_{3,3}$ and has as few edges as possible. Prove that *G* is simple and 3-connected.

SECTION – C (3 X 20 = 60)

ANSWER ANY THREE QUESTIONS

13. (i) Prove that a graph is bipartite if and only if it contains no odd cycle. (10)

    (ii) Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G. (10)

14. (i) With usual notations, Prove $k \leq \kappa' \leq \delta$. (10)

    (ii) In a bipartite graph, Prove that the number of edges in a maximum matching is equal to the number of vertices in a minimum covering. (10)

15. (i) State and prove Brooks' theorem in vertex colouring. (10)

    (ii) State and prove Vizing's theorem in edge colourings. (10)

16. (i) Explain stereographic projection in graph embedding with a diagram. (5)

    (ii) State and prove Five colour theorem. (15)

17. (i) Define Hypercube using binary sequences with a diagram. (5)

    (ii) State properties of Kautz network $K(d, n)$. (5)

    (iii) Explain topological structure of a circulant network with a diagram. State its properties and its applications. (10)

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