STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 and thereafter)

SUBJECT CODE : 19MT/PC/FA34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : FUNCTIONAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 × 2 = 10)**

1. State the Jensen's inequality.
2. State the closed graph theorem
3. Define the "Dual" of a space X.
4. State the parallelogram law for inner products on a liner space X.
5. Define "Normal" and "Self adjoint" operators.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 × 6 = 30)**

6. Let X be a normal space, If E_1 is open in X and $E_2 \subset X$ then prove $E_1 + E_2$ is open in X.
7. State and prove Hahn - Banach extension theorem.
8. Let X be a separable normed space. Then prove that every bounded sequence in X' has a weak convergent subsequence.
9. Let X be a normal space and $A \in BL(X)$. Then prove that A is invertible if and only if A is bounded below and surjective.
10. State and prove the Schwarz inequality for inner products on a linear space X.
11. Let H be an Hilbert space and $A, B \in BL(H)$ if A and B are normal such that A commutes with B^* and B commutes with A^* then prove that $A + B$ and AB are normal.
12. State and prove Bessel's inequality for countable orthonormal sets in an inner product space X.

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SECTION – C

ANSWER ANY THREE QUESTIONS: (3 × 20 = 60)

13. Let X be a normal space. Then prove that the following statements are equivalent.

    (i) Every closed and bounded subset of X is compact.

    (ii) the subset $\{x \in X : \|x\| \leq 1\}$ of X is compact

    (iii) X is finite dimensional.

14. State and prove open - mapping theorem.

15. Let X be normed space and Let E be a suspect of X then E is bounded in X if and only if $F(E)$ is bounded in K for every $f \in$ in X', Where X' is dual of normed space X.

16. State and prove Gram - Schmidt orthonormalization process.

17. Let H be a Hilbert space and $A, B \in BL\ (H)$

    (a) If A and B are self - adjoint then prove $A + B$ is self adjoint. Also prove AB is self adjoint if A and B are commute.

    (b) Let (A_n) be a sequence in $BL(H)$ and $A \in BL(H)$ such that $\|A_n - A\| \to 0$ as $n \to \infty$. If each A_n is self-adjoint, unitary or normal then prove that A is self-adjoint, unitary or normal respectively.