STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20)

SUBJECT CODE : 19MT/PC/AA14

M. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : ABSTRACT ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(5 × 2 = 10)**
ANSWER ALL QUESTIONS

1. List all the conjugate classes in S_3. Find the c_a's.
2. Find all the units of the ring Z[i], of all Gaussian integers.
3. Prove that the ring R[x], of all polynomials in *x* over an integral domain *R* is an integral domain.
4. If L is an extension field of a field F of degree 7 and if K is a field such that $F \subseteq K \subseteq L$ then what will be the possible degrees of K over F?
5. Prove that the symmetric group S_4 is solvable.

SECTION – B **(5 × 6 = 30)**
ANSWER ANY FIVE QUESTIONS

6. Prove that any group of order $11^2 . 13^2$ is abelian.
7. Define a principal ideal in a ring and prove that any ideal in a Euclidean ring is a principal ideal.
8. State and prove the Eisenstein Criterion about the irreducibility of a polynomial with integer coefficients.
9. Define the splitting field of a polynomial *f(x)* over a field F. Find the splitting field of the polynomial $f(x) = x^3 - 2$ over the field of rational numbers. Also find its degree over Q, the field of rational numbers.
10. Prove that a group *G* is solvable if and only if $G^{(k)} = (e)$ for some integer *k*.
11. Let G be a group and suppose that G is the internal direct product of N_1 , N_2 , ... , N_n. Let T = $N_1 \times N_2 \times \dots \times N_n$. Prove that the groups G and T are isomorphic
12. Prove that any element in a Euclidean ring R is either a unit or can be uniquely written as a product of finite number of prime elements of R up to associates.

SECTION – C **(3 × 20 = 60)**

ANSWER ANY THREE QUESTIONS

13. (a) Prove that the number of p-Sylow subgroups in G, for a given prime, is of the form 1+kp.
    (b) Prove that any two p-Sylow subgroups of a finite group G are conjugates. (10 +10)

14. (a) Prove that the ring of Gaussian integers is a Euclidean ring.
    (b) Define greatest common divisor of any two elements in a commutative ring R. Prove that any two elements *a* and *b* in a Euclidean ring R has a greatest common divisor d. Moreover $d = \lambda a + \mu b$, for some $\lambda, \mu \in R$. (10 + 10)

15. (a) Define a primitive polynomial. If *f(x)*and *g(x)*are primitive polynomials, prove that *f(x)g(x)*is also a primitive polynomial.
    (b) If R is a unique factorization domain, prove that the polynomial ring R[x] is also a unique factorization domain, by stating all the results used.

16. (a) If $a, b \in K$ are algebraic over *F* of degrees *m* and *n* respectively and if m and n are relatively prime, prove that *F(a, b)* is of degree *mn* over F
    (b) Define an algebraic integer and an algebraic number. Also if a rational number r is an algebraic integer, prove that r must be an ordinary integer. (10 + 10)

17. Prove that K is a normal extension of F if and only if K is a splitting field of a polynomial over F.