STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE : 19MT/MC/VA53

B. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : VECTOR ANALYSIS AND APPLICATION
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10×2=20)**
ANSWER ANY TEN QUESTIONS

1. Prove that the derivative of a constant vector is zero vector.
2. Define Direction Derivative of a scalar function.
3. Write the Laplace's equation.
4. Define Flux of vector function $\vec{F}$ over a surface $\vec{S}$.
5. Define circulation.
6. State Greens theorem.
7. Prove that $div\, \vec{r} = 3$.
8. If $\vec{a}$ is a constant vector, find $curl(\vec{r} \text{ x } \vec{a})$.
9. For any two vectors $\vec{A}$ and $\vec{B}$, find $div\left(\vec{A} \text{ x } \vec{B}\right)$.
10. Find a unit vector which is normal to the surface $z = x^2 + y^2$ at the point $(1,2,5)$.
11. State the physical interpretation of *curl*.
12. Find the constant *a*, so that the vector $V = (2x + y)\vec{\imath} - (3y + 2z)\vec{\jmath} + (x + az)\vec{k}$.

SECTION – B **(5×8=40)**
ANSWER ANY FIVE QUESTIONS

13. If $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$, prove that i) $\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$, ii) $\nabla r^n = nr^{n-2}\vec{r}$.
14. If *f* and *g* are two scalar point functions, then i) $grad(fg) = f\ grad\ g + g\ grad\ f$ ii) $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$.
15. If $\vec{F} = x^2y\vec{\imath} - 2xz\vec{\jmath} + 2yz\vec{k}$, find $div\,\vec{F}, curl\,\vec{F}, curl\ curl\,\vec{F}$.
16. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\vec{\imath} - 5z\vec{\jmath} + 10z\vec{k}$, along the curve $x = t^2 + 1, y = 2t^2, z = t^3$, from $t = 1$ and $t = 2$.
17. Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{\imath} - yz^2\vec{\jmath} - y^2z\vec{k}$, where *S* is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and *C* is its boundary.

18. Find the direction derivative of $\phi = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ, where Q has coordinates $(5,0,4)$.
19. Discuss about physical interpolation of divergence.

SECTION – C **(2×20=40)**

ANSWER ANY TWO QUESTIONS

20. a) Show that $\nabla^2 \left(\frac{x}{r^3}\right) = 0$.

b) Find a unit vector which is normal to the surface $z = x^2 + y^2$ at the point $(1,2,5)$.

(10+10)

21. Verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{\imath} + (y^2 - zx)\vec{\jmath} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelopiped, $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.

22. Derive the equation to calculate divergence in terms of curvilinear co-ordinates.