STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE : 19MT/MC/RA55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : PRINCIPLES OF REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10 × 2 = 20)**
ANSWER ANY TEN QUESTIONS

1. Define : non decreasing function.
2. What is an open ball in R^1 ?
3. Define : Metric Space.
4. Define : Cauchy sequence in a metric space.
5. Define : Open set in a metric space.
6. What is a Complete metric space ?
7. State the generalization of the Nested Interval theorem.
8. Define : Bounded set.
9. Define : Compact metric space.
10. State the Heine Borel property.
11. State the Chain Rule for derivatives.
12. State Rolle's theorem.

SECTION – B **(5 × 8 = 40)**
ANSWER ANY FIVE QUESTIONS

13. If $lim_{x\to a} f(x) = L$ and $lim_{x\to a} g(x) = M$, prove that $lim_{x\to a} [\, f(x) + g(x) \,] = L + M$.
14. Let F be any nonempty family of open subsets of a metric space M. Prove that $\bigcup_{G \in F} G$ is an open subset of M.
15. If E is any subset of a metric space M, show that $\bar{E}$ is closed.
16. If the subset A of the metric space (M, ρ) is totally bounded, prove that A is bounded.
17. Given that the metric space M has the Heine-Borel property, prove that M is compact.

18. State and prove the law of the mean.
19. State and prove the Second fundamental theorem of calculus.

SECTION – C (2 × 20 = 40)
ANSWER ANY TWO QUESTIONS

20. (a) Let $\boldsymbol{f}$ and $\boldsymbol{g}$ be real valued functions . Given that f is continuous at a and g is continuous at $f(a)$, prove that $g \, o \, f$ is continuous at a.

(b) State and prove the Picard Fixed Point Theorem.

(10 + 10)

21. (a) Let (M_1 , ρ_1) and (M_2 , ρ_2) be metric spaces. Prove that $f : M_1 \to M_2$ is continuous on M_1 if and only if $f^{-1}(G)$ is open in M_1, whenever G is open in M_2.

(b) If A is a closed subset of a complete metric space $\langle M, \ \rho\rangle$, prove that $\langle A, \ \rho\rangle$ is complete. (12 + 8)

22. (a) Let $(M_1 , \ \rho_1)$ be a compact metric space. If f is a continuous function from M_1 into a metric space $(M_2 , \ \rho_2)$, prove that f is uniformly continuous on M_1 .

(b) If $f \in R[a,b]$, $g \in R[a,b]$, prove that $f + g \in R[a,b]$, and

$$\int_a^b f + g = \int_a^b f + \int_a^b g .$$

(10 + 10)

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