STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE : 19MT/MC/IT54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : INTEGRAL TRANSFORMS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10×2=20)**
ANSWER ANY TEN QUESTIONS

1. Define exponential order of a function.
2. Find $L(\sin^2 2t)$.
3. Find $L^{-1}\left[log\frac{s+1}{s-1}\right]$.
4. Write the sufficient conditions for the existence of the Laplace transformation.
5. Write the complex form of Fourier integrals.
6. Prove that if $F\{f(x)\} = F(s)$, then $F\{f(x-a)\} = e^{isa}F(s)$.
7. Write the Fourier transform pairs.
8. Find the z transform of $(-1)^n$.
9. Find the z transform of $e^{-3t}\ t$.
10. Prove that $Z(n) = \frac{z}{(z-1)^2}$.
11. Find the inverse Z-transform of $\frac{z}{(z-1)^2}$.
12. Solve the difference between $y_{n+1} - 5y_n = 0$, using Z-transforms.

SECTION – B **(5×8=40)**
ANSWER ANY FIVE QUESTIONS

13. Find $L^{-1}\left[\frac{7s-1}{(s+1)(s+2)(s+3)}\right]$.
14. Solve $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 5y = 4e^{3t}$ given that $y(0) = 2, y'(0) = 7$.
15. Determine y which satisfies the equation

    $\frac{dy}{dt} + 3y + 2\int_0^t y\,dt = t$ for which $y(0) = 0$.
16. Express the function $f(x) = \begin{cases} 1\ for\ |x| \le 1 \\ 0\ for\ |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate

    $\int_0^\infty \frac{\sin\lambda\cos\lambda}{\lambda}\,d\lambda$ and $\int_0^\infty \frac{\sin\lambda}{\lambda}\,d\lambda$.

17. Find the Fourier cosine transform of $f(x) = e^{\frac{-x^2}{2}}$.

18. Obtain the z- transform of $\cosh n\theta$.

19. Using the inversion integral method, find the z-transform of $\frac{3z}{(z-1)(z-2)}$.

SECTION – C (2×20=40)
ANSWER ANY TWO QUESTIONS

20. a. Using Laplace transformation solve the system of equations.

$$\frac{dy}{dt} = 2x - 3y$$

$$\frac{dy}{dt} = y - 2x$$

Given that $x(0) = 8, y(0) = 3$.

b. Evaluate $\int_0^\infty \frac{e^{-t}-e^{-2t}}{t}\, dt$.

21. a. Find the Fourier cosine integral of the function e^{-ax}. Hence deduce the value of the integral $\int_0^\infty \frac{\cos\lambda x}{1+\lambda^2}\, d\lambda$.

b. Show that $Z\left(\frac{1}{n!}\right) = e^{\frac{1}{z}}$. Hence evaluate $Z\left[\frac{1}{(n+1)!}\right]$.

22. a. If $F(z) = \frac{2z^2+3z+12}{(z-1)^4}$, find the value of $f(2)$ and $f(3)$.

b. Solve the difference equation

$$y_{n+3} - 6y_{n+2} + 12y_{n+1} - 8y_n = 1, y_0 = 1, y_1 = 1 , y_2 = 2.$$