STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2019–20 and thereafter)

SUBJECT CODE 19MT/MC/EG34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE	:	**MAJOR – CORE**
PAPER	:	**ELEMENTS OF GRAPH THEORY**
TIME	:	**3 HOURS** — **MAX. MARKS: 100**

SECTION-A

Answer any TEN questions **(10 x 2 = 20)**

1. Define bipartite graph.
2. Prove that the sum of the degrees of the points of a graph G is twice the number of lines.
3. Define incidence matrix of the graph.
4. Show that the partition $P(7,6,5,4,3,2)$ is not graphic.
5. Prove that if a graph G with at least two points is bipartite then all its cycles are of even length.
6. Define closure of a graph.
7. Prove that every Hamiltonian graph is 2-connected.
8. Let G be a (p, q) connected graph. Then show that $q \geq p - 1$.
9. Define elementary subdivision of a graph.
10. Prove that K_5 is not planar.
11. Define digraph.
12. What are the three types of connectivity in directed graph?

SECTION-B

Answer any FIVE questions **(5 x 8 = 40)**

13. Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.
14. Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph. Then prove that
    (i) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1p_2)$ graph
    (ii) $G_1 \times G_2$ is a $(p_1p_2, q_1p_2 + p_1q_2)$ graph.
15. In a graph G, prove that any $u - v$ walk contains a $u - v$ path.
16. Prove that $c(G)$ is well defined.
17. Show that every tree has a centre consisting of either one point or two adjacent points.

18. State and prove Euler's theorem.
19. Let A be the adjacency matrix of a graph G. Then prove that $a_n(i,j)$, the ij entry in the matrix A^n, gives the number of paths of length n from v_i to v_j.

SECTION-C

Answer any TWO questions (2 x 20 = 40)

20. (a) Prove that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

 (b) Show that a closed walk of odd length contains a cycle. (12+8)

21. (a) Let G be a connected graph with at least three points. Prove that the following statements are equivalent.

 (i) G is a block

 (ii) Any two points of G lie on a common cycle

 (iii) Any point and any line of G lie on a common cycle

 (iv) Any two lines of G lie on a common cycle.

 (b) If G is a graph in which the degree of every vertex is at least two then prove that G contains a cycle. (14+6)

22. (a) Write Warshall's Algorithm to find shortest path.

 (b) Let G be a (p,q) graph. Then prove that the following statements are equivalent.

 (i) G is a tree

 (ii) Every two points of G are joined by a unique path

 (iii) G is connected and $p = q + 1$

 (iv) G is acyclic and $p = q + 1$

(6+14)