STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2019–20 & thereafter)

SUBJECT CODE : 19MT/MC/DC14

B. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : DIFFERENTIAL CALCULUS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10 × 2 =20)**

ANSWER ANY TEN QUESTIONS

1. If $x = acos\theta$, $y = bsin\theta$, find $\frac{d^2y}{dx^2}$.
2. If $y = sin3xcos2x$, find y_n.
3. If $= \tan^{-1} x$, then prove that $(1 + x^2)y_1 = 0$.
4. Find the radius of curvature at any point (p, r) of the curve whose pedal equation is $p^2 = ar$.
5. State the formula for radius of curvature in parametric form.
6. Show that radius of curvature of the curve $y = ccosh\left(\frac{x}{c}\right)$ is $\frac{y^2}{c}$.
7. Define envelope.
8. Find the envelope of $xcos\alpha + ysin\alpha = \alpha$, α being the parameter.
9. State the necessary condition for maximum and minimum of extrema with two variables.
10. Find the stationary points of the function $x^2 + y^2 + (x + y + 1)^2$.
11. Give the equation of asteroid. State its characteristic property.
12. Define catenary.

SECTION – B **(5X8=40)**

ANSWER ANY FIVE QUESTIONS

13. If $y = \frac{x^2+x-1}{x^3+x^2-6x}$. Find y_n.
14. Find the chord of curvature through pole of the curve $r = a(1 + cos\theta)$.
15. Find the equation of circle of curvature at the point $(3,1)$ on the curve $y = x^2 - 6x + 10$.
16. Find the envelope of $y = mx + \sqrt{a^2m^2 + b^2}$.
17. Show that the function $f(x, y) = x^2 + 3x^2y + 5x$ has a minimum at $(0,0)$.
18. Derive the equation of a Cycloid.
19. Search for double points of the curve $x^4 + y^3 + 2x^2 + 3y^2 = 0$.

SECTION – C **(2 × 20 = 40)**

ANSWER ANY TWO QUESTIONS

20. a) If $y = \sin(m \sin^{-1} x)$ then show that

$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0.$

b) Find the evolute of the curve $x = a(cost + tsint)$, $y = a(sint - tcost)$.

21. a) Find the envelope of straight line $\frac{x}{a} + \frac{y}{b} = 1$, where the parameter a and b are connected by the relation $ab = c^2$.

b) Find the evolute of parabola $y^2 = 4ax$.

22. a) Find the minimum value of $x^2 + y^2 + z^2$, subject to the condition

$2x + 3y + 5z = 30.$

b) Examine the character of the origin on the curve $y^2 = 2x^2y + x^3y + x^3$.