STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019–20 & thereafter)

SUBJECT CODE :19MT/MC/AS55

B. Sc. DEGREE EXAMINATION, NOVEMBER 2022
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRAIC STRUCTURES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

Answer any ten questions: **(10 × 2 =20)**

1. Give one reason why the set of all 2 x 2 matrices with real entries is not a group.
2. Find the subgroup generated by 5 in the group Z_{30}.
3. What is a permutation group? Give an example.
4. There is no isomorphism from the group of rational numbers under addition to the group of nonzero rational numbers under multiplication. Why?
5. Define automorphism of a group. Illustrate with an example.
6. Let $H = \{0, 3, 6\}$ in Z_9 under addition. Enumerate the cosets of H in Z_9.
7. Show that every subgroup of an abelian group is normal.
8. The mapping ϕ from $\boldsymbol{R^*}$ to $\boldsymbol{R^*}$ is defined by $\phi(x) = |x|$. Find the kernel of this homomorphism.
9. Define a ring.
10. What is a ring homomorphism?
11. Define the symmetry group of a figure in $\boldsymbol{R^n}$.
12. Find the characteristic of a ring with unity.

SECTION – B

Answer any five questions: **(5 × 8 = 40)**

13. Show that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.
14. Prove that every group is isomorphic to a group of permutations.
15. Let ϕ be a homomorphism from a group G to a group $\bar{G}$ and let H be a subgroup of G. Prove that (i) If H is normal in G, then $\phi(H)$ is normal in $\phi(G)$ (ii) If $|H| = n$, then $|\phi(H)|$ divides n.
16. State and prove Two-step subgroup test.

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17. Prove that a finite integral domain is a field.
18. Let R be a commutative ring with unity and Let A be an ideal of R. Show that R/A is an integral domain if and only if A is prime.
19. Let D be an integral domain. Prove that there exists a field F that contains a subring isomorphic to D.

SECTION – C

Answer any two questions: **($2 \times 20 = 40$)**

20. Define a cyclic group. Give an example. State and prove the Fundamental theorem of cyclic groups.
21. (i) State and prove Fermat's Little theorem.

    (ii) Let G be a group and let H be a normal subgroup of G. Prove that the set $G/H = \{aH/a \in G\}$ is a group under the operation $(aH)(bH) = abH$.
22. (i) State and prove the theorem on existence of Factor rings.

    (ii) Show that the dihedral group D_n is the plane symmetry group of a regular n – gon.

    (iii) Prove that the only finite plane symmetry groups are $\boldsymbol{Z_n}$ and $\boldsymbol{D_n}$.

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