

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2019-2020 and thereafter)

SUBJECT CODE : 19MT/AC/ST35

B. Sc. DEGREE EXAMINATION, NOVEMBER 2022

BRANCH I - MATHEMATICS

THIRD SEMESTER

COURSE : ALLIED – CORE

PAPER : MATHEMATICAL STATISTICS - I

TIME : 3 HOURS

MAX. MARKS : 100

(10X2=20)

SECTION – A

ANSWER ANY TEN QUESTIONS

1. A random variable X has the following probability function. Find k .

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

2. Define probability function.
3. The joint probability distribution of two random variables X and Y is given by $P(X = 0, Y = 1) = \frac{1}{3}, P(X = 1, Y = -1) = \frac{1}{3}, P(X = 1, Y = 1) = \frac{1}{3}$. Find the Marginal Distribution of X and Y
4. A person draws 2 balls from a bag containing 2 red and 3 black balls. He will get Rs. 20, for every red ball drawn by him and Rs. 10, for every black ball drawn, what is the amount he can expect to get?
5. Define Moment Generating Function of a random variable.
6. The number of monthly breakdowns of a computer is a R.V. having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month.
(a) Without a breakdown (b) With at least one breakdown.
7. Two dice are thrown 120 times. Find the average number of times in which the number on the first dice exceeds the number on the second dice.
8. Define Normal Distribution.
9. If X and Y are independent random variables each following normal distribution with mean 0 and standard deviation 3. What will be the mean and standard deviation for the random variable $3X + 4Y$.
10. Define Correlation.
11. Find the rank correlation for a set of 10 observations given that the sum of squares of the difference in their ranks is 122.5
12. The Coefficient of correlation between two variate X and Y is 0.48. Their covariance is 36. The variance of X is 16. Find the standard deviation of Y .

SECTION – B
ANSWER ANY FIVE QUESTIONS

(5X8=40)

13. If a random variable X takes the values 1, 2, 3 and 4 such probability that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$, find the probability distribution and cumulative distribution of X .
14. Compute the standard deviation and mean deviation about mean of the continuous probability distribution whose probability density function is given by $f(x) = 6x(1 - x)$ in $0 \leq x \leq 1$.
15. If (X, Y) is a two-dimensional continuous random variable with joint probability density function $f(x, y)$ then prove that (i) $E(X + Y) = E(X) + E(Y)$ and (ii) $E(XY) = E(X)E(Y)$ where X and Y are independent random variables.
16. Fit a binomial distribution for the following data:

x	0	1	2	3	4	5	6	Total
f	5	18	28	12	7	6	4	80

17. The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and S.D. 5. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75?
18. Derive moment generating function and the values of the first three central moments of the Poisson distribution.
19. Find the rank correlation coefficient for the following data:

X	48	33	40	9	16	16	65	24	16	57
Y	13	13	24	6	15	4	20	9	6	19

SECTION – C
ANSWER ANY TWO QUESTIONS

(2X20=40)

20. a) State and prove Bayes' theorem for future events.
- b) From a vessel containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and is found to be white. What is the probability that out of four balls transferred 3 are white and 1 is black?

(10 + 10)

21. a) The following is the cumulative distribution function of a discrete random variable X :

x	-3	-1	0	1	2	3	5	8
$F(x)$	0.10	0.30	0.45	0.65	0.75	0.90	0.95	1.00

i) Find the probability distribution of X , ii) $P(X \text{ is even})$ and $P(1 \leq X \leq 8)$ and

iii) $P(X = -3 / X < 0)$ and $P(X \geq -3 / X > 0)$

b) Prove that the Normal distribution is the limiting form of Binomial distribution.

(10 + 10)

22. a) If X and Y are independent Poisson random variables such that

$$\text{Var}(X + Y) = 9 \text{ and } P\{X = 3 | (X + Y = 6)\} = \frac{5}{54} \text{ find } E(X)$$

b) The following table gives the frequency according to age groups of marks obtained by

67 students in an intelligence test

Marks	Age in years				Total
	18	19	20	21	
100-250	4	4	2	1	11
250-300	3	5	4	2	14
300-350	2	6	8	5	21
350-400	1	4	6	10	21
Total	10	19	20	18	67

Is there any relationship between age and intelligence?

(8 + 12)

