STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/PE/CI15

M. Sc. DEGREE EXAMINATION, APRIL 2022
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : CALCULUS OF VARIATION AND INTEGRAL EQUATIONS
TIME : 3hours **MAX. MARKS: 100**

Section – A
Answer ALL questions (5 × 2 = 10)

1. Find the Extremal of $\int_{x_0}^{x_1}(x + y')y'dx$
2. Derive the Euler's Equation for functional which linear in y'.
3. Define Resolvent Kernel for Fredholm integral equation with an example.
4. Brief on the method to find the Iterated Kernels
5. List out the application of Green's functions

Section – B
Answer any FIVE questions (5 × 6 = 30)

6. Derive the Euler equation for the functional $I[y(x)] = \int_a^b F(x, y, y')dx$ with $y(a) = A, y(b) = B$.
7. Determine the extremal of $I[y(x)] = \int_0^\pi((y')^2 - y^2)dx$ with $y(0) = 0, y(\pi) = 1$ and subject to $\int_0^\pi ydx = 1$.
8. Find the shortest distance between A(1,0) and the ellipse $4x^2 + 9y^2 = 16$.
9. Reduce the initial Value problem $y'' - 5y' + 6y = 0, y(0) = 0, y'(0) = -1$ into Voltera integral equation of second kind.
10. Evaluate $y(x) = 2x - \pi + 4f(x) + \int_0^{\frac{\pi}{2}} sin^2x\, y(t)dt$
11. Obtain the resolvent kernel for $y(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xt\, y(t)dt$
12. Construct Green's function for $y'' = 0, y(0) = y(l) = 0$.

Section – C
Answer any THREE question (3 × 20 = 60)

13. a) State and prove Euler's Ostrogradsky equation.

b) Find the Extremal of the functional $I[y(x)] = \int_{x_0}^{x_1}\left(y''^2 - 2(y')^2 + y^2 - 2y\, sinx\right)dx.$

(10+10)

14. a) Derive the transversality condition to find the extremal for functional with moving end Points.

b) Find the shortest distance between the parabola $y = x^2$ and the line $y = x - 5.$ (8+12)

15. a) Using successive approximation method evaluate $y(x) = 1 + \int_0^x (x - t)y(t)dt.$

b) Solve $y(x) = x + \lambda \int_0^1 (xt^2 - x^2t)y(t)dt.$ (10+10)

16. a) Find the approximate solution of the integral equation $g(s) = 1 + \lambda \int_0^1 (s + t)g(t)dt.$

b) State and prove Fredholm Integral theorem. (10+10)

17. Using Green's function obtain the solution for $y'' - y = x, y(0) = y(1) = 0;$

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