STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/PC/TO24

M. Sc. DEGREE EXAMINATION, APRIL 2022
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE: CORE
PAPER : TOPOLOGY
TIME: 3 HOURS **MAX MARKS:100**

SECTION-A (5×2=10)

ANSWER ALL THE QUESTIONS

1. Define a topological space.
2. Give an example of a connected space.
3. Prove that the real line $\mathbb{R}$ is not compact.
4. Define Regular and Normal spaces.
5. Explain product topology.

SECTION-B (5 × 6 = 30)

ANSWER ANY FIVE QUESTIONS

6. Let Y be a subspace of X . Prove that a set A is closed in Y if and only if it is equal to the intersection of a closed set of X with Y.
7. Prove that the collection $S = \{\pi_1{}^{-1}(U)/U\ open\ in\ X\} \cup \{\pi_2{}^{-1}(V)/\ V\ open\ in\ Y\}$ is a subbasis for the product topology on $X \times Y$.
8. Prove that a space X is locally connected if and only if for every open set U of X ,each component of U is open in X.
9. Prove that the image of a connected space under a continuous map is connected.
10. State and prove Extreme value theorem.
11. Show that every compact Hausdorff space is normal.
12. State and prove Pasting lemma.

SECTION-C (3 × 20 = 60)

ANSWER ANY THREE QUESTIONS

13. a) Prove that the collection of all subsets of a set whose complement is either finite or the whole set is a topology.

b) Prove that arbitrary union and finite intersections of open sets are open.

c) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

(6 + 7 + 7)

14. a) If S denote the following subset of the plane: $S = \{x \times \sin(1/\, x)/0 < x \leq 1\}$, then prove that $\bar{S}$ is not path connected.

b) Prove that the union of a collection of connected subspaces of X that have a point in common is connected.

c) Prove that a finite cartesian product of connected spaces is connected.

(6 + 6 + 8)

15. If X is a metrizable space, then prove that the following are equivalent:

i) X is compact.

ii) X is limit point compact.

iii) X is sequentially compact.

16. State and prove Urysohn Metrization Theorem.

17. If X and Y are topological spaces and $f: X \to Y$, then prove that the following statements are equivalent:

i) f is continuous.

ii) For every subset A of X, $f(\bar{A}) \subset \overline{f(A)}$.

iii) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

iv) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

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