STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/PC/LA24

M. Sc. DEGREE EXAMINATION, APRIL 2022
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : CORE
PAPER : LINEAR ALGEBRA
TIME : 3 HOURS **MAX. MARKS : 100**

Section-A
Answer ALL the questions **(5x2=10)**

1. If the subspace M of dimension 'm', of a vector space V is cyclic with respect to T, then show that the dimension of MT^k is m-k for all $k \leq m$.
2. Write down the basic Jordan block belonging to the characteristic value λ.
3. Prove that similar matrices have the same characteristic polynomial.
4. Prove that if T and U are linear operators on V, then $(TU)^*=U^*\, T^*$.
5. Define sesqui-linear form.

Section-B
Answer any FIVE questions **(5x6=30)**

6. Prove that two nilpotent linear transformations are similar if and only if they have the same invariants.
7. Write down all the possible Jordan form for a 6×6 matrix with the minimal polynomial $x^2(1-x)^2$.
8. If $T: V \rightarrow W$ is a linear transformation, then prove that dim V = rank T + nullity T.
9. Let V and W be finite dimensional inner product spaces over the same field F. Then, prove that V and W are isomorphic if and only if they have the same dimension.
10. Let V be a complex vector space and f be a form on V such that $f(\alpha, \alpha)$ is real for all α. Then, prove that f is Hermitian.
11. Let V be a finite dimensional inner product space, T- a linear operator on V and B be an orthonormal basis for V. Suppose that the matrix A of T in the basis B is upper triangular then prove that T is normal iff A is a diagonal matrix.
12. Find the characteristic values and characteristic vector corresponding to the characteristic value for the matrix $\begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$. Also find the rank and nullity of the corresponding characteristic space.

Section-C

Answer any THREE questions **(3x20=60)**

13. If $T \in A(V)$ has all its characteristic roots in F then obtain the triangular form of the matrix of T.

14. a) For each i = 1, 2, …, k, prove that $V_i \neq \{0\}$ and $V = V_1 \oplus V_2 \oplus \dots \oplus V_k$. Also, prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$ where $q_i(x)$ are distinct irreducible polynomials.

b) Prove that every linear transformation $T \in A_F(V)$ satisfies its characteristic polynomial and every characteristic root of T is a root of $p_T(x)$.

(15+5)

15. a) State and prove Cayley-Hamilton theorem.

b) Find the minimal polynomial for the matrix $\begin{pmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{pmatrix}$.

(14+6)

16. a) Prove that for every invertible complex $n\times n$ matrix B, there exists a unique lower triangular matrix M with positive entries on the main diagonal such that MB is unitary.

b) Let V be a finite dimensional inner product space and T be any linear operator on V. Suppose W is a subspace of V invariant under T, then prove that the orthogonal complement of W is invariant under T^*.

(14+6)

17. State and prove the principal axis theorem along with the supporting Lemma(s).