STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE: 19MT/PC/DG44

M.Sc. DEGREE EXAMINATION, April 2022
BRANCH I – MATHEMATICS
FOURTH SEMESTER

TITLE: DIFFERENTIAL GEOMETRY
CORE: CORE
TIME: 3 HOURS **MAX: 100 MARKS**

SECTION – A

Answer all the questions (5 × 2 = 10)

1. Find the Cartesian equation of the parametrized curve $\vec{\gamma}(t) = (e^t,\ t^2)$.
2. Compute the first fundamental form of the surface patch $\vec{\sigma}(u, v) = (\cosh u, \sinh u\,, v)$.
3. Define an orientable surface.
4. State Meusnier's theorem.
5. Define Gaussian and mean curvatures of a surface patch.

SECTION – B

Answer any five questions (5 × 6 = 30)

6. If $\vec{\gamma}(t)$ is a regular curve in $\mathbb{R}^3$ then derive the equation of curvature.
7. Compute the curvature and torsion for the curve $\vec{\gamma}(t) = \left(\frac{4}{5}\cos t\,, 1 - \sin t\,, -\frac{3}{5}\cos t\right)$.
8. Prove that the unit sphere in $\mathbb{R}^3$ is a surface.
9. Prove that any tangent developable is isometric to (part of) a plane.
10. Prove that every plane in $\mathbb{R}^3$ is a surface with an atlas containing a single surface patch.
11. State and prove Euler's theorem.
12. Prove that a curve on a surface is a geodesic if and only if its geodesic curvature is zero everywhere.

SECTION – C

Answer any three questions (3 × 20 = 60)

13. State and prove the fundamental theorem on uniqueness and existence of space curves.

14. a) Discuss whether the Mobius band is orientable.

    b) Compute the Gaussian curvature and mean curvature of the elliptic paraboloid

    $\vec{\sigma}(u, v) = (u - v, u + v, u^2 + v^2)$. (8 + 12)

15. a) Prove that a diffeomorphism $f : S_1 \to S_2$ is an isometry if and only if, for any surface patch $\bar{\sigma}_1$ on S_1, the first fundamental forms of $\bar{\sigma}_1$ and $f \circ \bar{\sigma}_1$ are the same.

b) Prove that the area of a surface patch is unchanged by reparametrisation. (12 + 8)

16. a) Prove that the normal curvature of a unit speed curve on a surface patch $\bar{\sigma}$ is the second fundamental form of $\bar{\sigma}$.

b) Let κ_1 and κ_2 be the principal curvatures at a point P of a surface patch $\bar{\sigma}$. Then prove that

(i) κ_1 and κ_2 are real numbers

(ii) if $\kappa_1 = \kappa_2 = \kappa$, then $F_{II} = \kappa F_I$ and every tangent vector to $\bar{\sigma}$ at P is a principal vector:

(iii) if $\kappa_1 \neq \kappa_2$, then any two (non-zero) principal vectors $\bar{t}_1$ and $\bar{t}_2$ corresponding to κ_1 and κ_2, respectively are perpendicular. (7 + 13)

17. State and prove Gauss's remarkable theorem.

****