STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)
SUBJECT CODE: 19MT/PC/CF44
M.Sc. DEGREE EXAMINATION, April 2022
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : CORE
PAPER : CONTINUUM AND FLUID MECHANICS
TIME : 3 HOURS **MAX. MARKS : 100**

Section – A
Answer **ALL** questions $(5 \times 2 = 10)$

1. Suppose that the equations $a_{ip}a_{jp}c_j = b_{ij}c_j$ hold for arbitrary c_i, then show that $a_{ip}a_{jp} = b_{ij}$.
2. Determine the dyadic $\bar{G} = \bar{D} \cdot \bar{F}$, if $\bar{D} = 3\hat{\imath}\hat{\imath} + 2\hat{\jmath}\hat{\jmath} - \hat{\jmath}\hat{k} + 5\hat{k}\hat{k}$ and $\bar{F} = 4\hat{\imath}\hat{k} + 6\hat{\jmath}\hat{\jmath} - 3\hat{k}\hat{\jmath} + \hat{k}\hat{k}$.
3. If the velocity $\bar{q} = x\hat{\imath} - y\hat{\jmath}$, then determine the equation of the stream lines.
4. Discuss the pressure at a point in rest.
5. Prove that if $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x, y)$ at all points (x, y) of a region S in the plane Ox, Oy bounded by a closed curve C and if f is prescribed at each point (x, y) of S and at each point of C, then any solution $w = w(x, y)$ satisfying these conditions is unique.

Section – B
Answer **ANY FIVE** questions $(5 \times 6 = 30)$

6. In the x_i system, a vector $\bar{a}$ has components $a_1 = -1,\ a_2 = 0,\ a_3 = 1$ and a tensor $\bar{A}$ has its matrix $[a_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$. The $x_i{}'$ system is obtained by rotating the x_i system about the x_3 axis through an angle of 45° in the sense of the righthanded screw. Find the components of $\bar{a}$ and the matrix $\bar{A}$ in the $x_i{}'$ system.
7. Prove the vector identity $(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) = (\bar{a} \cdot \bar{c})(\bar{b} \cdot \bar{d}) - (\bar{a} \cdot \bar{d})(\bar{b} \cdot \bar{c})$ using the permutation symbol.
8. A displacement field is given by $\bar{u} = X_1X_2\hat{e_1} + X_1{}^2X_2\hat{e_2} + X_2{}^2X_3\ \hat{e_3}$. Determine independently the mateial deformation gradient $\bar{F}$ and the material displacement gradient $\bar{J}$ and verify the relation between the two.

9. Liquid flows through a pipe whose surface is the surface of revolution of the curve $y = a + k\,x^2/a$ about the x −axis $(-a \leq x \leq a)$. If the liquid enters at the end $x = -a$ of the pipe with velocity V, show that the time taken by a liquid particle to traverse the entire length of the pipe from $x = -a$ to $x = a$ is $\frac{2a}{V(1+k)^2}\left(1 + \frac{2}{3}k + \frac{1}{5}k^2\right)$.
10. Derive Bernoulli's equation for potential flow under conservative body forces and deduce the equation for incompressible and steady fluid flow.
11. Discuss the flow for which $w = z^2$.
12. Discuss the steady flow through tube of uniform circular cross-section.

Section – C

Answer **ANY THREE** questions $(3 \times 20 = 60)$

13. Discuss the three invariants of a second order tensor and prove that for a skew tensor $\bar{A}$, $I_{\bar{A}} = III_{\bar{A}} = 0$ and $II_{\bar{A}} = |\omega|^2$, where ω is the dual tensor of $\bar{A}$.

14. (a) Derive the relation between the stress vector and stress tensor.
    (b) Discuss the stress quadric of Cauchy. (12+8)

15. (a) Derive the equation of continuity of the fluid flow.
    (b) Discuss the acceleration of a fluid. (12+8)

16. (a) Discuss the complex potential for line sources and line sinks.
    (b) Find the equations of the streamlines due to uniform line sources of strength m through the points $A(-c, 0), B(c, 0)$ and a uniform line sink of strength $2m$ through the origin. (10+10)

17. Derive the Navier-Stokes equations of motion of a viscous fluid. Also discuss the particular cases.

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