

M.Sc. DEGREE EXAMINATION, April 2022
BRANCH I – MATHEMATICS
FOURTH SEMESTER

TITLE: COMPLEX ANALYSIS

CORE: CORE

TIME: 3 HOURS

MAX: 100 MARKS

SECTION – A

Answer ALL the questions ($5 \times 2 = 10$)

1. Show that $\int_{\gamma} (z - a)^n dz = 0$, for closed curve γ and $n \geq 0$. Explain the case when $n = -1$.
2. Construct an analytic function from Harmonic function.
3. Establish the relation between the zeta function and prime numbers.
4. Define equicontinuity.
5. What is an analytic arc? When is it regular or simple?

SECTION – B

Answer ANY FIVE questions ($5 \times 6 = 30$)

6. Prove that the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$ for a piecewise differentiable closed curve γ that do not pass through the point a .
7. State and prove the Mean value property for Harmonic functions.
8. Derive the Poisson's formula.
9. Obtain the Legendre's duplication formula from $\Gamma(z)\Gamma\left(z + \frac{1}{2}\right) = e^{az+b}\Gamma(2z)$, where a and b are constants.
10. Evaluate the values of Zeta function at negative integers and zero.
11. Explain the necessary and sufficient conditions for a family of functions to be totally bounded.
12. Discuss the behaviour at an angle in the transformation of polygon onto unit disc.

SECTION – C

Answer ANY THREE questions ($3 \times 20 = 60$)

13. State and prove Cauchy's theorem for a rectangle and a disc.
 14. Define Harmonic function and prove all its properties.
 15. Derive the expression $\frac{\pi}{\sin \pi z} = \sum \frac{(-1)^n}{z-n}$.
 16. State and prove the Riemann mapping theorem.
 17. Discuss the fluid flow in a channel through a slit and with an offset.
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