STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/MC/VL64

B. Sc. DEGREE EXAMINATION, APRIL 2022
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : VECTOR SPACES AND LINEAR TRANSFORMATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY TEN QUESTIONS. **(10×2=20)**

1. Define linear dependence and independence of vectors in a vector space.
2. Why the set $\{(1,3),(4,1),(1,1)\}$ cannot be a bases for the vector space R^2?
3. Prove that the set U of 2×2 diagonal matrices is a subspace of the vector space M_{22} of 2×2 matrices.
4. Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 0 & 1 & 2 \end{bmatrix}$.
5. State Gram –Schmidt Orthogonalization process.
6. Define kernel and range of a linear transformation.
7. Show that the transformation $T: R^2 \to R^2$ defined by $T(x, y) = (2x, x + y)$ is linear.
8. Define reflection.
9. Find the coordinate vector $\bar{u} = (4, 5)$ relative to the basis $B = \{(2, 1), (-1, 1)\}$.
10. Define similarity transformation.
11. Define inner product on a real vector space.
12. Show that the functions $f(x) = 3x - 2$ and $g(x) = x$ are orthogonal in P_n with inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$.

SECTION –B

ANSWER ANY FIVE QUESTIONS. **(5×8=40)**

13. Prove if $\{\overline{v_1}, \overline{v_2}, \dots, \overline{v_n}\}$ is a basis for a vector space V, then $\{\overline{w_1}, \overline{w_2}, \dots, \overline{w_m}\}$ a set of more than n vectors in V is linearly dependent. Also prove that all bases for a vector space have the same number of vectors.
14. a) State and prove Orthogonal matrix theorem. b) The vectors $(1, 0, 0)$, $\left(0, \frac{3}{5}, \frac{4}{5}\right)$, $\left(0, \frac{4}{5}, -\frac{3}{5}\right)$ form a basis for R^3then express the vector $(7, -5, 10)$ as a linear combination of these vectors. (5+3)

15. Construct an orthonormal basis for the set of vectors $\{(1,\ 2,\ 0,\ 3),\ (4,\ 0,\ 5,\ 8),\ (8,\ 1,\ 5,\ 6)\}$ that form a basis for a three dimensional space V of R^4.

16. Prove that for a linear transformation $T: U \to V$, the kernel of T is a subspace of U and range of T is a subspace of V.

17. Prove that the linear transformation $T: R^n \to R^n$ is invertible if and only if it is non singular. Also prove the inverse is unique and linear.

18. Define transition matrix and find the transition matrix from a basis $B = \{(1,2),(3,-1)\}$ to $B' = \{(3,1),(5,2)\}$ of R^2. Also if $\bar{u}$ is a vector such that $\bar{u}_B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, find $\bar{u}_{B'}$.

19. Define angle between two vectors and determine the angle between the functions $f(x) = 5x^2$ and $g(x) = 3x$ in P_n with inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$.

SECTION –C

ANSWER ANY TWO QUESTIONS. **(2×20=40)**

20. a) Define a vector space and prove that C^n is a vector space.
    b) Find the basis for the subspace V of R^4 spanned by the vectors $(1,2,3,4)$, $(-1,-1,-4,-2)$, $(3,4,11,8)$. (14+6)

21. a) State and prove rank or nullity theorem.
    b) If $T: R^3 \to R^2$ is a linear transformation defined by $T(x,y,z) = (x+y,\ 2z)$, then find the matrix of T with respect to the bases $\{(1,1,0),(0,1,4),(1,2,3)\}$ and $\{(1,0),(0,2)\}$. Also find the image of the vector $(2,3,5)$. (10+10)

22. a) Define matrix representation of a linear transformation $T: U \to V$ with respect to the bases $B = \{\overline{u_1}, \dots, \overline{u_n}\}$ and $B' = \{\overline{u_1'}, \dots, \overline{u_n'}\}$. Also, prove that if $\bar{u}$ is a vector in U with image $T(\bar{u})$, having coordinate vectors $\bar{a}$ and $\bar{b}$ relative to the bases then $\bar{b} = A\bar{a}$.
    b) Prove that $\langle \bar{u}, \bar{v}\rangle = ae + 2bf + 3cg + 4dh$ is an inner product where $\bar{u} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\bar{v} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ be arbitrary elements of M_{22}. Also determine the inner products of the pair of matrices $\bar{u} = \begin{bmatrix} -2 & 4 \\ 1 & 0 \end{bmatrix}$, $\bar{v} = \begin{bmatrix} 5 & -2 \\ 0 & -3 \end{bmatrix}$ (10+10)