STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/MC/SS44

B. Sc. DEGREE EXAMINATION, APRIL 2022
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : SEQUENCE AND SERIES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY TEN QUESTIONS: (10×2=20)

1. Define composition of functions and find $g \circ f$ if $f(x) = x + 7$ and $g(x) = 2x$ where $(x \in I)$.
2. What is a Cantor set?
3. Define least upper bound and give an example of a set which has an upper bound but not a least upper bound.
4. Define subsequence of real numbers and hence find a subsequence of $C = \{c_n\}_{n=1}^{\infty} = \{\sqrt{n}\}_{n=1}^{\infty}$, if the subsequence of positive integers is defined by $N = \{n_i\}_{i=1}^{\infty} = \{i^4\}_{i=1}^{\infty}$.
5. Can a subsequence of a divergent sequence converge? Justify with a suitable example.
6. Define limit superior of a sequence of real numbers.
7. Discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(1-n)}{(1+2n)}$.
8. Define absolute convergence and conditional convergence.
9. Give an example of a series which dominates the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ with justification.
10. Prove that the series $\sum_{n=1}^{\infty} \frac{2n}{(n^2-4n+7)}$ diverges.
11. Define Fourier series.
12. Prove that $\int_{-a}^{a} f(x)\, dx = 0$, if $f(x)$ is odd.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5×8=40)

13. Prove that the countable union of countable sets is countable.
14. Prove that the limit of a convergent sequence is unique.
15. Prove that $\{x^n\}_{n=1}^{\infty}$ converges to 0, if $0 < x < 1$ and diverges if $1 < x < \infty$.
16. If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of real numbers, then prove that $\limsup_{n\to\infty} s_n = \lim_{n\to\infty} s_n$.
17. Prove that if $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ converges.
18. State and prove comparison test and use it to show that the geometric series $\sum_{n=0}^{\infty} x^n$ converges absolutely for any $x \in (-1, 1)$.
19. Expand $f(x)$ as Fourier series in the interval $-\pi$ to π if $f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2×20=40)**

20. a) Prove that the inverse image of the union of two sets is the union of the inverse images.

    b) State and prove Ratio test. (8+12)

21. a) State and prove Nested interval theorem.

    b) Find the limit of the sequence $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. (8+12)

22. Show that in the range 0 to 2π, the expansion of e^x as a Fourier series is

$$e^x = \frac{e^{2\pi}-1}{\pi}\left\{\frac{1}{2}+\sum_{n=1}^{\infty}\frac{\cos nx}{n^2+1}-\sum_{n=1}^{\infty}\frac{\text{n}\sin nx}{n^2+1}\right\}.$$