STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2019-20 Thereafter)

SUBJECT CODE : 19MT/MC/IC23

B. Sc. DEGREE EXAMINATION, APRIL 2022
BRANCH I – MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : INTEGRAL CALCULUS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY TEN QUESTIONS: **(10X2=20)**

1. Evaluate $\int \frac{dx}{\sqrt{x}+\sqrt{1+x}}$
2. Evaluate $\int \frac{x}{\sqrt{1-x^2}}dx$
3. Evaluate $\int \frac{dx}{\sqrt{x^2-2x+5}}$
4. Which of the following integral is improper? Give reason.
   a. $\int_{-\infty}^{\infty} \frac{\sin x}{1+x^2}dx$ b. $\int_1^2 \frac{1}{2x-1}dx$
5. State the comparison test for improper integral.
6. Define the Gamma function.
7. Prove that $\beta(m,n) = \beta(n,m)$.
8. Evaluate $\int_0^a \int_0^b xy\, dx\, dy$.
9. State Fubini's theorem.
10. Using polar coordinates, evaluate $\iint_D x\, dA$, where D is the disk with centre the origin and radius 3.
11. Find the Jacobian of x, y with respect to u, v if $x = u + v,\ y = uv$.
12. If $x = r\cos\theta$ and $y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5X8=40)**

13. Evaluate $\int_0^{\pi/2} \frac{dx}{9\cos x+12\sin x}$
14. Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx$
15. For what values of p is the integral $\int_1^{\infty} \frac{1}{x^p}$ convergent?

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16. Show that $\int_0^\infty xe^{-x^8}dx\ .\ \int_0^\infty x^2e^{-x^4}\ dx = \frac{\pi}{16\sqrt{2}}$.

17. Prove that $\frac{\beta(p,q+1)}{q} = \frac{\beta(p+1,q)}{p} = \frac{\beta(p,q)}{p+q}$.

18. Evaluate $\iint_R\ (x-y)dx\ dy$ where R is the region between the line $x = y$ and the parabola $y = x^2$.

19. Using change of variable to evaluate $\iint_R\ xy\ dx\ dy$, where R is the region in the first quadrant bounded by the hyperbolas $x^2 - y^2 = a^2$ and $x^2 - y^2 = b^2$ and the circles $x^2 + y^2 = c^2$ and $x^2 + y^2 = d^2$ $(0 < a < b < c < d)$.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2X20=40)**

20. Evaluate the following integrals

    a. $\int \frac{dx}{(1+x^2)\sqrt{(1-x^2)}}$

    b. $\int \sqrt{(x-3)(7-x)dx}$

    c. $\int \frac{sec^2x\ dx}{a^2cos^2x+b^2sin^2x}$ (8+8+4)

21. a) Prove that $\int_0^2 z^2 \ln z\ dz$ is convergent.

    b) State and prove the relation between the Beta and Gamma function. (8+12)

22. a) Evaluate the iterated integral $\int_0^1 \int_x^1 \sin y^2\ \ dy\ dx$ by reversing the order of integration.

    b) Using change of variable evaluate $\iint_R\ (x-y)^4\ e^{x+y}\ dx\ dy$, where R is the square with vertices $(1,0), (2,1), (1,2)$ and $(0,1)$. (10+10)