STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

SUBJECT CODE : 19MT/MC/DM43

B. Sc. DEGREE EXAMINATION, APRIL 2022
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE : MAJOR CORE
PAPER : DISCRETE MATHEMATICS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ANY TEN QUESTIONS: **(10×2=20)**

1. Define Logical equivalence of propositions.
2. Define Contrapositive proposition and give the contrapositive of the statement: "If John is a poet, then he is poor".
3. Define universal quantifier and determine the truth value of the statement $\exists\, x\, \forall y,\ x^2 < y + 1$, where $U = \{1, 2, 3\}$ is the universal set.
4. Define Bounded Lattice.
5. Draw the Hasse diagram of $L_1 \times L_2$, where L_1 and L_2 be the chains of divisors of 4 and 9.
6. Define join irreducible and give example.
7. Show that $(a')' = a$, where a is an element of a Boolean algebra.
8. Find the consensus Q of $P_1 = xyz's$ and $P_2 = xy't$.
9. Define a Finite State Machine.
10. Design a finite state automaton that accepts precisely those strings over $\{a, b\}$ that contains an odd number of a's.
11. Define concatenation of languages and find the concatenation of $L_1 = \{a, b^3\}$ and $L_2 = \{a^3, ab^2, b\}$.
12. Define Kleene closure of a Language.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5×8=40)**

13. Write the condition for an argument to be valid and check the validity of the argument
$p \to \neg q, r \to q, r \vdash \neg p$.
14. State and prove the idempotent property and associative property of a Lattice.
15. If $(L, \leq)$ is a lattice and $a \leq b \leq c$, where $a, b, c \in L$, then show that
(i) $a \vee b = b \wedge c$.
(ii) $(a \wedge b) \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.

(4+4)

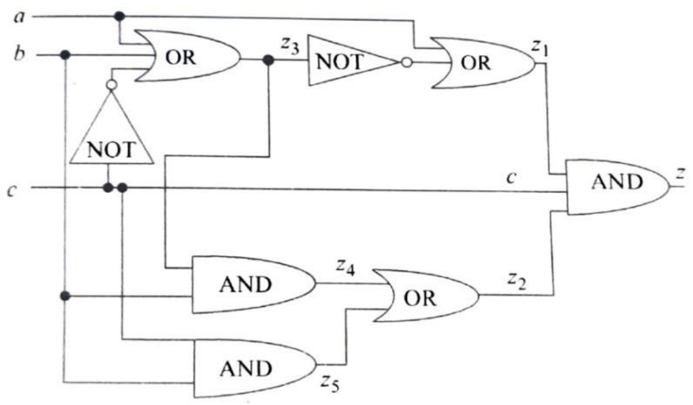
16. Find the complete sum of product form for the following expressions:

(i) $x_1 \vee x_{2.}$

(ii) $x(y'z)'$ **(4+4)**

17. a) Draw the logic circuit for $ab' + a'b$.

b) Find the Boolean expression for the following logic diagram:



(3+5)

18. Draw the transition diagram for the finite state machine $M(I, S, O, s_o, f, g)$, where $I = \{a, b\}, S = \{s_0,\ s_1, s_2\}, O = \{x, y, z\}$, Initial state: s_0.Next state function $f: S \times I \to S$ defined by $f(s_0, a) = s_1,\ f(s_1, a) = s_2,\ f(s_2, a) = s_0,\ f(s_0, b) = s_2,\ f(s_1, b) = s_1,$ $f(s_2, b) = s_1$Output function $g: S \times I \to O$ defined by $g(s_0, a) = x,\ g(s_1, a) = x,$ $g(s_2, a) = z,\ g(s_0, b) = y,\ g(s_1, b) = z,\ g(s_2, b) = y.$ Also, find the output string for the input string $abaab$.

19. a) Define regular expression and language over a finite state of symbols.

b) Find a regular expression *r* such that $L = \{a^m b^n : m, n > 0\}$ be a language over $A = \{a, b\}$.

(4+4)

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2×20=40)**

20. a) Find the conjunctive normal form and disjunctive normal form of $p \leftrightarrow (\neg p \vee \neg q)$ using the laws of algebraic propositions.

b) If $(L_1, \leq)$ and $(L_2, \leq)$ are distributive lattices, then prove that $(L, \leq)$, where $L = L_1 \times L_2$ is a distributive lattice. **(10+10)**

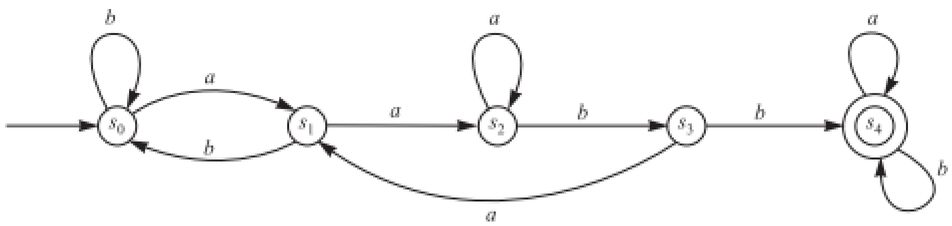
21. a) State and prove De Morgan's law for a Boolean algebra. Also show the uniqueness of complement in a Boolean algebra.

b) Draw the transition table and transition Diagram for FSA, $M = \{I, S, A, s_0, f\}$, where $I = \{0,1,2,3,4,5,6,7,8,9\}$, $S = \{s_0, s_1, s_2\}$, $A = \{s_0\}, a \in \{0, 3,6,9\}$, b $\in \{1,4,7\}$, $c \in \{2,5,8\}$, next state function defined by $f(s_0, a) = s_0$, $f(s_0, b) = s_1, f(s_0, c) = s_2, f(s_1, a) = s_1$, $f(s_1, b) = s_2, f(s_1, c) = s_0$, $f(s_2, a) = s_2, f(s_2, b) = s_0, f(s_2, c) = s_1$. Does this automaton accept 672 and 7348?

(12+8)

22. a) Find the prime implicants and minimal sum of product form for the expression $E(x, y, z) = xyz + x'z' + xyz' + x'y'z + x'yz'$ by consensus method.

b) Determine the language accepted by the automaton shown in the transition diagram:



(12+8)

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