STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019–20 & thereafter)

SUBJECT CODE : 19MT/MC/CA65

B. Sc. DEGREE EXAMINATION, APRIL 2022
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : PRINCIPLES OF COMPLEX ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ANY TEN QUESTIONS: **10 × 2 = 20**

1. Are analytic function differentiable? justify the converse?
2. Check whether the function $3x^2y + 2x^3 - y^3 - 2y^2$is harmonic or not.
3. Define conformal mapping for analytic function and check the transformation $w = \bar{\bar{z}}$ is conformal.
4. Find the critical point of the function $z + \frac{1}{z}$.
5. Obtain the Maclaurin series for cos z.
6. Compare Taylor's and Laurent's series.
7. Classify the singularity of the function $f(z) = (z - i)\sin\left(\frac{1}{z+2i}\right)$ and classify for them.
8. Write down Cauchy's inequality for analytic functions?
9. Find the fixed points of the transformations of $\frac{z+1}{1-z}$.
10. State and prove fundamental theorem of algebra.
11. Determine the zeros and poles of the analytic function $\frac{(z+1)^2(iz+2)^3}{z+7}$
12. Evaluate $\int_C \frac{dz}{z^2+4}$, where C is $|z - i| = 2$.

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 × 8 = 40**

13. Show that $u = \log\sqrt{x^2 + y^2}$ is harmonic and determine its conjugate and hence find the corresponding analytic function $f(z)$.
14. Find the bilinear transformation that maps
$z_1 = -i, z_2 = 0, z_3 = i$ into $w_1 = -1, w_2 = i, w_3 = 1$.
15. Discuss the applications of $w = e^z$ in upper half of the complex plane and check the mapping is conformal.
16. Find the series expansion using Taylors theorem for $\frac{z^2-1}{(z+2)(z+3)}$, $|z| < 2$.

17. Prove that $f'(z) = \frac{1}{2\pi i}\int_C \frac{f(\varsigma)d\varsigma}{(\varsigma - \mathrm{z})^2}$, where $f(z)$ is analytic function inside and on a simple closed curve C with z as any point inside C.
18. Evaluate $\int_0^{2\pi} \frac{d\theta}{2+cos\theta}$.
19. State and prove Rouche theorem and find the residue of $\int_C \frac{3z-4}{z(z-1)}$, where C: $|z| = 2$.

SECTION-C

ANSWER ANY TWO QUESTIONS: **2 × 20 = 40**

20. a) State and prove the necessary and sufficient condition for a given function to be analytic in a given region R.
    b) Prove that $Log(1-i) = \frac{1}{2}In\,2 - \frac{\pi}{4}i.$ (12+8)

21. a) State and prove Cauchy Goursat Theorem.
    b) If $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ find Laurent's series expansion in the region
    (i) $0 < |z-1| < 4$ (ii) $|z-1| > 4.$ (12+8)

22. a) State and prove Cauchy Integral formula and find $\int_C \frac{e^{2z}dz}{(z-1)^4}$, where C is $|z| = \frac{3}{2}$.
    b) Prove that $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}dx = \frac{5\pi}{12}$. (12+8)