

STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI - 86.
(For candidates admitted from the academic year 2019 and thereafter)

SUBJECTCODE: 19MT/PC/OD14

M.Sc DEGREE EXAMINATION, NOVEMBER 2021
BRANCH I- MATHEMATICS

COURSE: CORE

TIME: 3 Hours

PAPER: **ORDINARY DIFFERENTIAL EQUATIONS**

MAX. MARKS: 100

Section – A

Answer all the questions (2 × 4 =8)

1. Find the second approximation of $x_1(t)$ for initial value problem $x' = x^2$, $x(0) = 1$.
2. Find e^{At} when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

Section – B

Answer any two questions (2× 12 = 24)

3. State and prove Abel Formula.
4. Show that the coefficient of x^n in $P_n(x)$ is $\frac{(2n)!}{n!2^{2n}}$.
5. Find eigen values and eigen functions of the BVP $y' + \lambda y = 0$, $x(0) = 0$, $x'(1) = 0$.

Section – C

Answer any two questions (2 × 34= 68)

6. a. Find the power series solution for the equation $x''(t) + K\sin x(t) = 0$ with $x(0) = \frac{\pi}{6}$, $x'(0) = 0$.
b. Determine the fundamental matrix for the system $x' = Ax$ where $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 5 \end{bmatrix}$.
c. Prove that $J_{-n}(x)$ is not independent of $J_n(x)$ and evaluate $J_{-1/2}(x)$, $J_{3/2}(x)$.
(12+10+12)
7. a. State and prove Picard Existence and uniqueness theorem for the IVP $\frac{dy}{dx} = f(x, y)$; $y(x_0) = y_0$.
b. Find the $W[y_1, y_2]$ for $y'' + 2xy' + 2y = 0$, $y_1(0) = 0$, $y_2(0) = 1$, $y_1'(0) = -1$, $y_2'(0) = 1$ (20+14)
8. a. Define Green's function and prove the necessary and sufficient condition for any function to be the solution of non-homogenous BVP using Green's function.
b. Obtain the Green's function for $y'' + k^2y = 0$ with condition $y(0) = y(1) = 0$.
c. State and prove the orthogonal property of Sturm Liouville problem.
(10+10+14)
