STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086

(For candidates admitted from the academic year 2019 – 20)

SUBJECT CODE: 19MT/PC/PD34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2021

BRANCH I - MATHEMATICS

THIRD SEMESTER

COURSE : CORE

PAPER : PARTIAL DIFFERENTIAL EQUATIONS

TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER **ALL** THE QUESTIONS (2 × 4 = 8)

1. Can the partial differential equation $yp - xq = 0$ be obtained from the equation $x^2 + y^2 = (z - c)^2 tan^2 \alpha$ (where c and α are constants)? Justify!
2. Write a homogenous wave equation and classify it!

SECTION – B

ANSWER ANY **TWO** QUESTIONS (2 × 12 = 24)

3. Describe Cauchy's problem for first order equations briefly.
4. Reduce the following equation to a canonical form:

$$(1 + x^2)^2 u_{xx} + (1 + y^2)^2 u_{yy} + x u_x + y u_y = 0$$

5. Solve the one-dimensional diffusion equation in the region $0 \le x \le \pi$, $t \ge 0$ subject to the conditions (i) T remains finite as $t \to \infty$ (ii) $T = 0$, if $x = 0$ and $x = \pi$ for all t

(iii) At $t = 0,\ T = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi. \end{cases}$

SECTION – C

ANSWER ANY **TWO** QUESTIONS (2 × 34 = 68)

6. a) Find the surface which intersects the surface of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$. (12)

   b) Does the transformation of independent variables of partial differential equation modify classification of that partial differential equation? Justify your answer! (10)

   c) Find a complete integral of $x^2p^2 + y^2q^2 - 4 = 0$ using Charpit's method. (12)

7. a) Find the solution of interior Dirichlet problem for a circle. (14)

b) Derive Laplace's Equation. (10)

c) Find the D'Alembert's solution of a Cauchy type problem involving one-dimensional wave equation. Analyse the situation when the string is released from rest. (10)

8. a) Find the solution of one-dimensional wave equation by canonical reduction. (14)

b) In a one-dimensional infinite solid, $-\infty < x < \infty$, the surface $a < x < b$ is initially maintained at temperature T_0 and at zero temperature everywhere outside the surface. Show that $T(x,t) = \frac{T_0}{2}\left[erf\left(\frac{b-x}{\sqrt{4\alpha t}}\right) - erf\left(\frac{a-x}{\sqrt{4\alpha t}}\right)\right]$ where erf is an error function. (10)

c) For any continuous function $f(t)$, prove that $\int_{-\infty}^{\infty} \delta(t-a)f(t) = f(0)$ and (10) $\int_{-\infty}^{\infty} \delta(t-a)f(t) = f(a)$ where $\delta(t)$ is Dirac-delta function.
