STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)
SUBJECT CODE: 19MT/PC/AA14
M.Sc. DEGREE EXAMINATION, November 2021
BRANCH I – MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : **ABSTRACT ALGEBRA**
TIME : 3 hours MAXIMUM MARKS : 100

Section – A
Answer ALL questions ($2 \times 4 = 8$)

1. Let G be a group such that $o(G) = 9000$, find the order of subgroups which G certainly contains.
2. Show that $\sqrt{3} + \sqrt[3]{5}$ is algebraic over the field of rationals $\mathbb{Q}$.

Section – B
Answer ANY TWO questions ($2 \times 12 = 24$)

3. Prove that if G is a finite group, then the number of elements conjugate to a in G is the index of normalizer of a in G.
4. State and prove Eisenstein criterion.
5. Prove that if L is an algebraic extension of K and if K is an algebraic extension of F, then L is an algebraic extension of F.

Section – C
Answer ANY TWO questions ($2 \times 34 = 68$)

6. (a) Prove that if A and B are finite subgroups of G, then $o(AxB) = \frac{o(A)o(B)}{o(A \cap xBx^{-1})}$.

   (b) Prove that the number of p −Sylow subgroups in G equals $\frac{o(G)}{o(N(P))}$, where P is any p −Sylow subgroup of G.

   (c) Prove that if G and G' are isomorphic abelian groups, then for every integer $s, G(s)$ and $G'(s)$ are isomorphic.

   (10+14+10)

7. (a) State and prove the unique factorization theorem.

   (b) Let p be a prime integer and suppose that for some integer c relatively prime to p we can find integers x and y such that $x^2 + y^2 = cp$. Then prove that p can be written as the sum of squares of two integers.

   (c) Check whether the polynomial $x^2 + x + 1$ is reducible over F, the field of integers modulo 2.

   (12+12+10)

8. (a) Prove that if the field F is of characteristic zero and if a, b are algebraic over F, then there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

   (b) State and prove the fundamental theorem on Galois theory.

   (9+25)

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