STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86

(For candidates admitted during the year 2019 and thereafter)

SUBJECT CODE: 19MT/MC/VA53

B.Sc. DEGREE END SEMESTER EXAMINATION- NOVEMBER 2021

COURSE: MAJOR CORE **TIME: 3 hours**

PAPER: VECTOR ANALYSIS AND APPLICATIONS **MAX.MARKS: 100**

Section A

Answer all questions (3 × 4 = 12)

1. If $\frac{d\vec{a}}{dt} = \vec{\omega} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{\omega} \times \vec{b}$, show that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{\omega} \times (\vec{a} \times \vec{b})$.
2. Briefly explain the geometrical significance of the gradient $\nabla\phi$.
3. Show that $\int_S \nabla\phi . curl\vec{F}\, dV = \int_S (\vec{F} \times \nabla\phi). dS$.

Section B

Answer any three questions (3 × 16 = 48)

4. Find (i) $\frac{d^2}{dt^2}\left[\vec{r}, \frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}\right]$ (ii) $\frac{d}{dt}\left[\vec{r} \times \left(\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right)\right]$ (iii) $\frac{d}{dt}\left[\vec{r}, \frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}\right]$. (6+5+5)
5. Define curl of a vector point function and show that (i) $\nabla^2\left(\frac{x}{r^3}\right) = 0$ (ii) *curl grad* $\phi = 0$. (2+7+7)
6. Explain the divergence of a vector point function in terms of curvilinear co-ordinates.
7. Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative force field. Find the scalar potential. Find also the work done in moving an object in this field from (1, −2, 1) to (3, 1, 4). (4+5+7)

Section C

Answer any one question (1 × 40 = 40)

8. a) A particle moves along the curve $x = t^3 + 1,\ y = t^2,\ z = 2t + 5$, where *t* is the time. Find the components of its velocity and acceleration at *t* = 1 in the direction $\vec{i} + \vec{j} + 3\vec{k}$. Also find the unit tangent vector at any point to the curve.

   b) Verify Stoke's theorem for $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$, where *S* is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and *C* is its boundary.

   c) Show that (i) $\nabla.(\vec{A} \times \vec{B}) = \vec{B}.(\nabla \times \vec{A}) - \vec{A}.(\nabla \times \vec{B})$ (ii) $\nabla(\vec{A}.\vec{B}) = (\vec{B}.\nabla)\vec{A} + (\vec{A}.\nabla)\vec{B} + \vec{B} \times (\nabla \times \vec{A}) + \vec{A} \times (\nabla \times \vec{B})$. (14+12+14)

9. a) If $\vec{A}$ is a differentiable vector function and ϕ is a differentiable scalar function, then show that *div* $(\phi\vec{A})$ = *grad* $\phi . \vec{A} + \phi$ *div* $\vec{A}$.

   b) Find the equation of the tangent plane to the surface $xyz = 4$ at the point (1, 2, 2) on it.

c) Evaluate $\iint_S \vec{F} \cdot \vec{n}\ dS$, where $\vec{F} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

d) State Gauss divergence theorem. Verify divergence theorem for

$$\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$$

taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.

(6+6+13+15)