STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI - 86.
(For candidates admitted from the academic year 2019 and thereafter)

SUBJECTCODE: 19MT/MC/RA55

B.Sc DEGREE EXAMINATION, NOVEMBER 2021
BRANCH I- MATHEMATICS

COURSE: CORE **TIME: 3 Hours**
PAPER: PRINCIPLES OF REAL ANALYSIS **MAX MARKS: 100**

Section – A
Answer all the questions (3 × 4 = 12)

1. Prove that $\lim_{x\to 1}\frac{x^2-1}{x-1}=2$.
2. Show that the [0, 1] is not connected subset of R_d.
3. Verify $\int_a^\infty \frac{x}{1+x^2}dx$ is convergent or divergent

Section – B
Answer any Three questions (3 × 16 = 48)

4. Prove that every open set G of R^1 can be written $\boldsymbol{G}=\cup \boldsymbol{I_n}$ where $I_1, I_2, \ldots$ are mutually disjoint open intervals.
5. State and prove the generalized nested interval theorem on complete metric space.
6. Prove that the metric space $\langle M,\rho\rangle$ is compact if and only if every sequence of points in M has a subsequence convergent to a point in M.
7. Verify Cauchy mean value theorem for $f(x)=sinx,\ g(x)=cosx,\ \frac{-\pi}{2}\le x\le 0$.

Section – C
Answer any one question (1 ×40= 40)

8. a. Prove that the real valued function f is continuous at $a\ \epsilon\ R'$ iff the inverse image under f of any open ball $B[f(a),r]$ about $f(a)$ contains open ball $B[a,\delta]$ about a.

   b. Prove that the metric space with Heine Borel property is compact.

   c. If $\langle M_1,\rho_1\rangle$ and $\langle M_2,\rho_2\rangle$ be two metric and $f: M_1\to M_2$ then prove that f is continuous on M_1 if and only if $f^{-1}(F)$ is closed in M_1 when F is closed in M_2

   (12+15+13)

9. a. If a function is continuous in a compact metric space then prove that the function is uniformly continuous.

   b. State and Prove the necessary and sufficient condition for a function to be Riemann integrable

   c. State and prove the second Fundamental theorem of calculus. (10+15+15)

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