

STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86

(For candidates admitted during the year 2019 -2020 & thereafter)

SUBJECT CODE: 19MT/MC/IT54

B.Sc. DEGREE END SEMESTER EXAMINATION

BRANCH I- MATHEMATICS

FIFTH SEMESTER

COURSE: MAJOR CORE

PAPER : INTEGRAL TRANSFORM

MAX.MARKS: 100

TIME : 3 HOURS

SECTION-A

Answer **ALL** the questions

(3×4 = 12)

1. Find $L\{f(t)\}$, where $f(t) = 0$ when $0 < t \leq 2$
 $= 3$ when $t > 2$.
2. Using Fourier integral representation, show that $\int_0^\infty \frac{\omega \sin x\omega}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}, x > 0$.
3. If $U(z) = \frac{2z^2+3z+4}{(z-3)^3}, |z| > 3$, Find u_1 and u_2 .

SECTION-B

Answer any **THREE** questions

(3×16 = 48)

4. (a) Using Laplace transforms evaluate $\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t} dt$.
(b) Find the inverse Laplace transform of $\frac{1}{s(s-a)}$
5. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ Hence prove that
 $\int_0^\infty \frac{\sin s-s \cos s}{s^3} \cdot \cos \frac{s}{2} ds = \frac{3\pi}{16}$.
6. Find the Z-transform of (a) $a^n \cosh n\theta$ and (b) $a^n \sinh n\theta$.
7. Find the inverse Z-transform of $F(z) = \frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{4})}$ using convolution theorem.

SECTION-C

Answer any **ONE** question

(1×40 = 40)

8. a) Find $L^{-1} \left[\frac{s^2-s+2}{s(s-3)(s+2)} \right]$
b) Solve the Differential equation $2 \left(\frac{dx}{dt} - 3 \frac{dy}{dt} \right) = t, \frac{d^2y}{dt^2} + x = 2y$ given that $x = 0, y = 0,$
 $\frac{dy}{dt} = 0$ when $t = 0$.
c) Find the inverse Z-transform of $\frac{2z(z^2-1)}{(z^2+1)^2}$ by Power Series Method.
(10 + 20 + 10)
9. a) Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$. Hence derive Fourier sine transform of
 $\phi(x) = \frac{x}{1+x^2}$.
b) Solve $y_{k+2} - 2y_{k+1} + y_k = 2^k$ with $y_0 = 2, y_1 = 1$ using Z-transform .
(20 + 20)