

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 and thereafter)
SUBJECT CODE: 19MT/MC/DC14

B. Sc. DEGREE EXAMINATION, NOVEMBER 2021
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE: MAJOR – CORE
PAPER : DIFFERENTIAL CALCULUS

TIME: 3 HOURS
MAX. MARKS: 100

SECTION – A
Answer all the questions

(3 × 4 = 12)

1. Find the n^{th} derivative of $y = \sin^2 x$.
2. State the sufficient condition for extrema of functions with two variables.
3. Write any two properties of a catenary.

SECTION – B
Answer any three questions

(3 × 16 = 48)

4. Find the n^{th} derivative of $\frac{x^2}{(x+1)^2(x+2)}$.
5. Find the radius of curvature at the origin for the curve $x^3 + y^3 - 2x^2 + 6y = 0$.
6. Find the envelope of $\frac{x}{l} + \frac{y}{m} = 1$, where l and m are parameters and related by $\frac{l}{a} + \frac{m}{b} = 1$, a and b are constants.
7. Find the extreme values of $yz + 3zx + 2xy$ where $x + y + z = 1$.

SECTION – C
Answer any one question

(1 × 40 = 40)

8. a) If $x = \sin \theta$, $y = \sin p\theta$ then prove that
 - (i) $(1 - x^2)y_2 - xy_1 + p^2y = 0$
 - (ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (p^2 - n^2)y_n = 0$
- b) Find the radius of curvature of the curves $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \pi/4$.
- c) Show that the function $(x + y)^4 + (x - 3)^6$ has a minimum at $(3, -3)$.

(16 + 14+10)

9. a) Find the centre of curvature of the curve $y = x^3 + 2x^2 + x + 1$ at the point $(0,1)$.
- b) Find the evolute of the curve $x = a\cos\phi$, $y = b\sin\phi$, by treating the evolute as the envelope of its normal.
- c) Determine the nature and the existence of the double points on the curve

$$x^4 + 4x^3 + 2y^3 + 4x^2 + 3y^2 - 1 = 0.$$

(10+14 +16)
