

**Stella Maris College (Autonomous), Chennai - 600 086**  
(For candidates admitted during 2019 academic year and thereafter)  
**B.Sc. Degree Examination, November 2021**

Code: 19MT/MC/AS55

**Algebraic Structures**

Max. Marks: 100  
Course: Major Core

Duration: 3 hours

**Section A**

Answer **all** the questions ( $3 \times 4 = 12$ )

1. Prove that in a group  $(ab)^2 = a^2b^2$  if and only if  $ab = ba$ .
2. Define stabilizer and orbit of a point.
3. Check whether the mapping  $\varphi$  from the rings  $\mathbb{Z}_4$  to  $\mathbb{Z}_{12}$  given by  $\varphi(x) = 3x$  is a ring homomorphism.

**Section B**

Answer **any three** questions ( $3 \times 16 = 48$ )

4. State and prove the one-step subgroup test.
5. Determine the order of the permutation  $(12)(134)(152)$ . Also find whether the permutation is even or odd.
6. State and prove Lagrange's theorem.
7. Let  $R = \left\{ \begin{bmatrix} a & a \\ b & b \end{bmatrix} : a, b \in \mathbb{Z} \right\}$ . Prove or disprove that  $R$  is a subring of  $M_2(\mathbb{Z})$ .

**Section C**

Answer **any one** question ( $1 \times 40 = 40$ )

8. a) State and prove Cayley's theorem.  
b) Let  $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} : a, b, d \in \mathbb{R}, ad \neq 0 \right\}$ . Is  $H$  a normal subgroup of  $GL(2, \mathbb{R})$ .  
c) Prove that a finite integral domain is a field. (15 + 15 + 10)
9. a) Prove that an integral domain can be imbedded into a field.  
b) Find all the subgroups of  $Z_{30}$ . (25 + 15)

\*\*\*\*\*