STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

SUBJECT CODE : 19MT/AC/ST35

B.SC. DEGREE EXAMINATION, November 2021
BRANCH I – MATHEMATICS
THIRD SEMESTER

COURSE : ALLIED CORE
PAPER : MATHEMATICAL STATISTICS - I
TIME : 3 hours MAXIMUM MARKS : 100

SECTION –A

Answer ***ALL*** the questions (3×4=12)

1. If a random variable X takes the values $1, 2, 3, 4$ such that the probability satisfies
$$2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$$
Find the probability distribution of X.
2. Derive the expression for covariance of the random variables X and Y in terms of their expectations and also discuss what happens if the two random variables are independent.
3. Explain positive and negative correlation, linear and non-linear correlation.

SECTION –B

Answer ***ANY THREE*** questions (3×16=48)

4. The joint probability density function of a two-dimensional continuous random variable (X, Y) is given by $f(x,y) = x^2y + \frac{y^2}{8},\ 0 \le x \le 1,\ 0 \le y \le 2$. Compute $(i)\ P(Y > 1),\ \ (ii)\ P\left(X < \frac{1}{2}\right)$ $(iii)\ P\left(Y > 1 \,/\, X < \frac{1}{2}\right)\ \ (iv)\ P\{(X+Y) \le 1\}$.
5. If the joint probability density function of a two-dimensional continuous random variable (X, Y) is given by $f(x,y) = 3(x+y)$ in $x > 0, y > 0$ and $x + y \le 1$ and $= 0$ elsewhere, then find $\mathrm{Var}(X), \mathrm{Var}(Y), \mathrm{Cov}(X,Y)$.
6. The mean weight of 500 male students at a certain college is $151lb$ and the standard deviation is $15lb$. Assuming that the weights are normally distributed, find how many students weigh
(a) between $120lb$ and $155lb$ (b) more than $185lb$.
7. Calculate the Pearson's coefficient of correlation from the following data using 44 and 26 respectively as the origin of x and y:

x	43	44	46	40	44	42	45	42	38	40	42	57
y	29	31	19	18	19	27	27	29	41	30	26	10

SECTION –C

Answer ***ANY ONE*** question (1×40=40)

8. (a) Find the mean, median and mode of the distribution with probability density function $f(x) = \sin x$ in $0 \leq x \leq \frac{\pi}{2}$.

   (b) If the probability of hitting a target is $\frac{1}{5}$ and if 10 shots are fired, what is the conditional probability of the target being hit at least twice, assuming that at least one hit has been already achieved?

   (c) Find the rank correlation from the sales and expenses of 10 firms as given below:

Sales(X)	50	50	55	60	65	65	65	60	60	50
Expenses(Y)	11	13	14	16	16	15	15	14	13	13

(15+10+15)

9. (a) Derive the mean deviation about the mean of the normal distribution $N(\mu, \sigma)$.

   (b) Calculate the Karl Pearson's coefficient of correlation between x and y from the bivariate sample of 140 pairs of x and y as distributed below:

x \ y	10-20	20-30	30-40	40-50
10-20	20	26	-	-
20-30	8	14	37	-
30-40	-	4	18	3
40-50	-	-	4	6

   (c) Derive the moment generating function and values of the first four central moments of the Poisson distribution.

(12+16+12)

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