COURSE    :  MAJOR CORE
PAPER     :  GRAPH THEORY
TIME      :  3 HOURS                                    MAX. MARKS: 100

### SECTION – A
**Answer ALL questions**                              **(2×4=8)**

1.  Explain Konigsberg bridge problem.
2.  Define independent domination number and find the same for $K_5$.

### SECTION – B
**Answer any TWO questions**                          **(2×12=24)**

3.  What is meant by incidence and adjacency matrix of a graph and what are the column sums of those matrices? Illustrate with an example.
4.  Prove that $\alpha + \beta = \gamma = \alpha' + \beta'$ where α, β, $\alpha'$ and $\beta'$ are the independence number and covering number with respect to vertices and edges.
5.  Explain the basic principles of network design.

### SECTION – C
**Answer any TWO questions**                          **(2×34=68)**

6.  a) Show that a graph is bipartite if and only if it contains no odd cycle.
    b) With usual notations prove that $\kappa \le \kappa' \le \delta$.
    c) State and prove Brook's theorem.                 **(8+13+13)**

7.  a) Prove that every planar graph is 5-vertex colourable.
    c) Let $G$ be a graph on $v-1$ edges. Then show that the following conditions are equivalent:
       i)  $G$ is connected
       ii) $G$ is acyclic
       iii) $G$ is a tree.                               **(17+17)**

8.  a) State and prove Vizing's theorem.
    b) If a graph $G$ is simple then prove that $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$.
    c) Draw the following graphs: (i) $K(2, 3)$   (ii) $B(2, 2)$    (iii) $G(8; \{1, 2, 3\})$
                                                        **(15+7+12)**

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