STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86

(For candidates admitted during the year 2019 and thereafter)

SUBJECT CODE: 19MT/PC/FA34

M.Sc. DEGREE END SEMESTER EXAMINATION- NOVEMBER 2021

COURSE: CORE **TIME: 3 hours**
PAPER: FUNCTIONAL ANALYSIS **MAX.MARKS: 100**

Section A

Answer all questions ($2 \times 4 = 8$)

1. What are Function spaces? Discuss the different norms induced on these spaces.
2. When is the linear space X uniformly convex? Define and prove.

Section B

Answer any two questions ($2 \times 12 = 24$)

3. Let X and Y be Banach spaces and $F: X \to Y$ be a closed linear map. Then prove that F is continuous.
4. Let X be a separable normed space. Prove that every bounded sequence in X' has a weak* convergent subsequence.
5. Illustrate with an example to show that Projection theorem and Riesz representation theorem do not hold for an incomplete inner product space.

Section C

Answer any two questions ($2 \times 34 = 68$)

6. a) Let X be a normed space. Prove that the following conditions are equivalent.
   (i) Every closed and bounded subset of X is compact.
   (ii) The subset $\{x \in X: \|x\| \leq 1\}$ of X is compact.
   (iii) X is finite dimensional.
   b) Define spectrum of a bounded operator $A \in BL(X)$ and prove the result when inclusions $\sigma_e(A) \subset \sigma_a(A) \subset \sigma(A)$ become equalities.
   c) Discuss on an application of the bounded inverse theorem. (10+14+10)

7. a) Let H be a nonzero Hilbert space over K. Then prove that H is linearly isometric to K^n for some n, or to l^2, H is separable and H has a countable orthonormal basis are equivalent.
   b) Prove that the new normed spaces constructed from the old normed spaces by considering subspaces, quotient spaces, product spaces and spaces of bounded linear maps are Banach spaces.
   c) Establish Schur's lemma which states that $x_n \xrightarrow{w} x$ in X if and only if $x_n \to x$ in X for the case when $X = l^1$. (12+14+8)

8. a) Let H be a Hilbert space and $A \in BL(H)$. Then prove that there is a unique $B \in BL(H)$ such that for all $x, y \in H$, $\langle A(x), y \rangle = \langle x, B(y) \rangle$.
   b) State and prove Riesz representation theorem.
   c) Define a positive operator and hence prove Generalized Schwarz inequality. (10+14+10)