STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86

(For candidates admitted during the year 2019 and thereafter)

SUBJECT CODE: 19MT/MC/EG34

B.Sc. DEGREE END SEMESTER EXAMINATION- NOVEMBER 2020

COURSE: MAJOR CORE **TIME: 90 MINUTES**

PAPER: ELEMENTS OF GRAPH THEORY **MAX.MARKS: 50**

SECTION –A

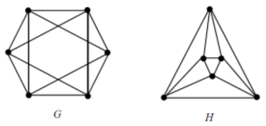
Answer all questions (3 × 2 = 6)

1. Define a cubic graph and prove that every cubic graph has an even number of points.
2. Give an example to show that the union of two distinct $u - v$ walks need not contain a cycle.
3. Prove that a finite directed graph G that is cycle-free contains a source and a sink.

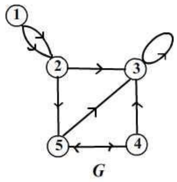
SECTION – B

Answer any three questions (3 × 8 =24)

4. (a) Define induced subgraph and automorphism of a graph G.
   (b) Show that the following graphs are isomorphic. (4+4)



5. (a) Let G be a connected graph with exactly $2n$ ($n \geq 1$), odd vertices. Then prove that the edge set of G can be partitioned into n open trails.
   (b) Prove that a line x of a connected graph G is a bridge if and only if x is not on any cycle of G. (4+4)
6. Prove that every polyhedron has at least two faces with the same number of edges on the boundary.
7. (a) Prove that every connected graph has a spanning tree.
   (b) Find the indegree and outdegree of the vertices in the given graph G. (5+3)



SECTION – C

Answer any one question (1 × 20 = 20)

8. (a) Prove that if A is the adjacency matrix of a graph with $V=\{v_1, v_2, \ldots, v_p\}$ prove that for any $n \geq 1$ the $(i, j)^{th}$ entry of A^n is the number of $v_i - v_j$ walks of length n in G.
   (b) State and prove Chavatal's theorem.
   (c) Let G_1 be a (p_1, q_1) graph and G_2 a (p_2, q_2) graph then show that $G_1 \times G_2$ is a $(p_1p_2, q_1p_2 + q_2p_1)$ graph. (7+8+5)
9. (a) Define centre of a tree and prove that every tree has a centre consisting of either one point or two adjacent points.
   (b) Describe Warshall's algorithm to find the path matrix P of the directed graph G and further explain the modified Warshall's algorithm to find the shortest path.
   (c) Define a graphic sequence and show that the partition P = (4,4,4,2,2,2) is graphical and hence construct graphs realizing the partitions. (5+10+5)