## STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-86

(For candidates admitted from the academic year 2019-20)
SUBJECT CODE: 19MT/AC/ST35

## B.Sc. DEGREE END SEMESTER EXAMINATION- NOVEMBER 2020

COURSE: ALLIED CORE
PAPER: MATHEMATICAL STATISTICS - I

TIME: 90 MINUTES
MAX.MARKS: 50

## SECTION - A

Answer ALL questions ( $3 \times 2=6$ )

1. If a card is drawn at random from a pack of cards. Find the probability that the card is either a spade or a king.
2. If X is a random variable with mean $\mu$ and standard deviation $\sigma$ then derive the variance of $z=\frac{x-\mu}{\sigma}$
3. Check whether the random variable $\mathrm{X}+\mathrm{Y}$ follows $B\left(n_{1}+n_{2}, p_{1}+p_{2}\right)$ when random variables $X \sim B\left(n_{1}, p_{1}\right)$ and $Y \sim B\left(n_{2}, p_{2}\right)$.

> SECTION - B

Answer any THREE questions ( $3 \times 8=24$ )
4. a) Define $\varphi_{X}(t)$, the characteristic function of a random variable X and prove that
$\mu_{r}^{\prime}=(-i)^{r}\left|\frac{\partial^{r}}{\partial t^{r}} \varphi_{X}(t)\right|$ at $\left.t=0 \mathrm{~b}\right)$ State and prove addition theorem of Expectation.
5. Derive mean, variance and moment generation function of Binomial distribution.
6. a) Define normal distribution. b) If $X$ is normally distributed and the mean of $X$ is 12 and variance is 16 , then find out the probability of i) $X \geq 20$ ii) $X \leq 20$ iii) $0 \leq X \leq 12$.
7. Calculate Karl Pearson's coefficient of correlation from the following data.

| X | 45 | 55 | 56 | 58 | 60 | 65 | 68 | 70 | 75 | 80 | 85 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 56 | 50 | 48 | 60 | 62 | 64 | 65 | 70 | 74 | 82 | 90 |

SECTION - C
Answer any ONE question ( $1 \times 20=20$ )
8. a) The joint density function of a random variable distribution is given by $f(x, y)=4 x y e^{-\left(x^{2}+y^{2}\right)}, x \geq 0, y \geq 0$. Find the marginal and conditional probability functions. Are X and Y independent?
b) i) The probabilities of a Poisson variate taking the values 3 and 4 are equal. Calculate the probabilities of the variable taking the values 0 and 2 .
ii) Derive the moments $\mu_{1}^{\prime}, \mu_{2}^{\prime}$ about origin of poisson distribution and hence find its mean and variance.
9. a) Prove that the normal distribution is a limiting case of the binomial distribution.
b) Ten competitors in a musical test were ranked by the three judges A, B and C in the
following order:

| Ranks by $A$ | 1 | 6 | 5 | 10 | 3 | 2 | 4 | 9 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ranks by B | 3 | 5 | 8 | 4 | 7 | 10 | 2 | 1 | 6 | 9 |
| Ranks by C | 6 | 4 | 9 | 8 | 1 | 2 | 3 | 10 | 5 | 7 |

Using rank correlation method, discuss which pair of judges has the nearest approach to common likings in music.


## Standard Normal Table (z)

Entries in the table give the area under the curve between the mean and $z$ standard deviations above the mean. For example, for $z=1.25$ the area under the curve between the mean (0) and $z$ is 0.3944 .

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0190 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2969 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3513 | 0.3554 | 0.3577 | 0.3529 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |

